

# Common Factors of CPI Sub-aggregates and Forecast of Inflation

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## Abstract

*In this paper, we investigate whether incorporating common factors of CPI sub-aggregates into forecasting models increases the accuracy of forecasts of inflation. We extract factors by both static and dynamic factor models and then embed them in ARMA and VAR models. Using quarterly data of Iran's CPI and its sub-aggregates, the models are estimated over 1990:2 to 2008:2 and out of sample forecasts are produced for 2008:3 to 2012:1. The results show that in most cases the performance of the models containing common factors of CPI sub-aggregates is better than the Autoregressive, as one of the benchmark models. But, only for the horizon of two-step ahead, the performance of the factor models are significantly better than that of benchmark. Also, the FAVAR performs better than the other factor models in forecasting inflation.*

**Keywords:** Forecasting, Inflation, CPI Sub-aggregates, Factor Models, ARMAX, FAVAR

**JEL Classification:** C32, C53, E31

## 1. Introduction

Since there are lags in transmission of monetary policy into the economy, the policy should be based on projections of the future of macro variables and particularly the future of inflation. Therefore, central banks around the globe employ inflation forecasting as a tool when setting monetary policy. Moreover, many economic decisions, made by firms, investors, or consumers, are often based on inflation forecasts. The accuracy of these forecasts can thus have important repercussions in the economy.

When forecasting aggregate economic variables consisting of several sub-aggregates, such as inflation, we face the question of, whether using information contained in sub-aggregates can improve the forecast accuracy of the aggregate variable. This is frequently encountered when forecasting inflation, where data are commonly available for components of CPI in addition to the CPI as an aggregate index (for more details, see Bermingham and D'Agostino, 2011).

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There are three ways for disaggregating CPI; we can disaggregate it (1) in terms of components of CPI (i.e. groups of goods and services), (2) in terms of regions or states, and (3) in terms of time (for more details, see Hendry and Hubrich, 2006). In this paper, we focus on the first one, i.e. disaggregating through components of CPI. When using information of components of CPI, we can consider three alternative approaches. The first one is modeling and forecasting components of CPI separately and then combining those forecasts to produce the forecast of CPI inflation (see Duarte and Rua, 2005, Espasa *et al.*, 2002, Hubrich, 2005, Bayat and Barakchian, 2013). In the second approach, introduced by Hendry and Hubrich (2006, 2011), some or all components are incorporated directly into the inflation forecasting model. Theoretically, including CPI sub-aggregates as exogenous variables in the forecasting model should improve forecast accuracy, but entering a great number of sub-aggregates in the model violates parsimony of the model, and because of generating estimation and specification errors, results in loss of efficiency (Lutkepohl, 2011). The third approach is based on using factor models to summarize information contained in CPI sub-aggregates. Since the factors extracted from CPI sub-aggregates have an effective role in explaining inflation (see, for example, Granger, 1987), it is plausible that those factors can improve forecast accuracy of inflation, while it does not violate the parsimony principle by adding too many parameters to the model. These factors can be embedded as exogenous variables in a vector ( $X$ ), and then be added to AR and ARMA models, resulting in ARX and ARMAX models, respectively. Mendez and Kapetanios (2005) used this approach for forecasting inflation of some countries in Euro-zone, and Duarte and Rua (2005) for forecasting inflation of Portugal. Factors can also be used in a Factor Augmented Vector Auto Regression (FAVAR) model to forecast inflation. The idea behind FAVAR is to combine the standard VAR analysis with the recent advances in dynamic factor models, and thus, to estimate a joint VAR including some variables of interest (here, inflation) plus factors extracted from a large panel of time series (here, CPI sub-aggregates). One of the advantages of the FAVAR model is that in a FAVAR, all variables are treated as endogenous, whereas in econometric modeling one generally needs to classify variables as exogenous and endogenous. The FAVAR model has been used frequently in academia and central banks for forecasting inflation (see, for example, Akdogan *et al.* 2012, Barnett *et al.* 2012, Faust and Wright, 2007, 2011, Pang, 2010, and Norman and Richard, 2010).

In this paper, we examine whether incorporating common factors of CPI sub-aggregates into a forecasting model improves its performance in forecasting inflation, and specifically, if such a model can outperform the autoregressive model as our simple but powerful benchmark model. To the best of our knowledge, no other study has yet examined performance of factor models in forecasting inflation in Iran. Moreover, this is the first study in which the information contained in CPI sub-aggregates is used to forecast inflation in Iran. (Bayat and Barakchian (2013) also use CPI sub-aggregates, however, the approach they follow is totally different from the one followed in this paper). In these respects, our study is perceived to be new. To compare the forecasting performance of different models, we use Root Mean Squared Forecast Error (RMSFE) as the measure. Our results show that in most cases the performance of models containing common factors of CPI sub-aggregates is better than that of the benchmark model. In addition, in the horizon of two-quarters ahead, the forecasts generated by the factor models are significantly more accurate than those generated by the benchmark model.

The remainder of this paper is organized as follows. Section 2 explains the factor models used in the paper. Section 3 presents the data and results, and finally, Section 4 concludes.

## 2. Factor Models

Common factors can be extracted from CPI components either by static or dynamic factor models. In this paper, the factors extracted by static factor models will be used in the class of ARMAX models and those extracted by dynamic factor models will be used in FAVAR models. The reason why factors in FAVAR models are extracted by dynamic factor models is that the setup of a FAVAR allows factors to be determined dynamically by their lags. This setup is compatible with state-space representation on which dynamic factor models are based.

### a. Static factor models

The standard formulation of a *static factor model* is

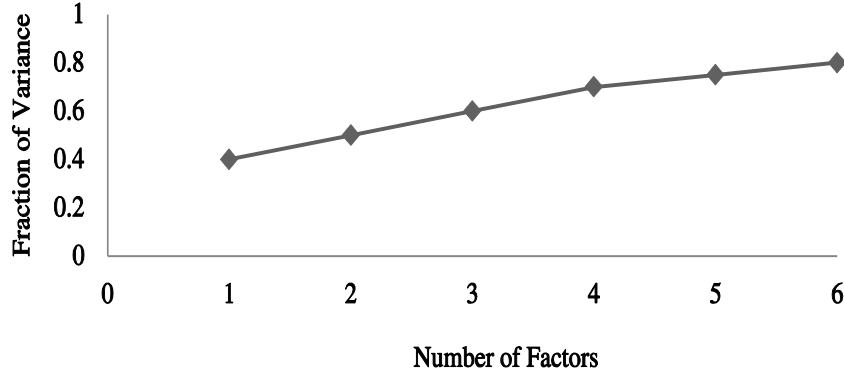
$$H_t = \Lambda F_t + e_t \quad (1)$$

Where  $H_t$  is a  $(N \times 1)$  vector of (the growth rates of) CPI sub-aggregates and  $F_t$  is a  $K \times 1$  vector of unobserved factors where  $K$  is small relative to  $N$ .  $\Lambda$  is a  $(N \times K)$  factor loading matrix,  $e_t$  is a  $(N \times 1)$  vector of errors following a multivariate normal distribution with zero mean and  $(N \times N)$  covariance matrix. For estimating factors, we use the method of maximum likelihood (MLE) which involves minimizing a discrepancy function. Let  $S$  represent the observed dispersion matrix of  $H$  (CPI sub-aggregates) and let the fitted dispersion matrix of  $H$  be  $\Sigma(\Lambda, \psi)$ . Then the discrepancy function, to be minimized by MLE, is

$$D_{MLE}(S, \Sigma) = tr[\Sigma^{-1} S] - \ln|\Sigma^{-1} S| - N.$$

But first we need to determine the number of factors before estimating them. Figure 1 shows the cumulative percentage of the total variation of (the growth rates of) the CPI sub-aggregates explained by the first 6 factors. As can be seen, with only 3 factors we are able to explain about 60% of the variation of (the growth rates of) the twelve CPI sub-aggregates (for the list of twelve CPI sub-aggregates, see footnote 1). So we assume that the number of factors is 3.

**Figure 1: Cumulative fraction of variance of CPI sub-aggregates explained by the factors**



Another rule for choosing the number of factors is called the Kaiser-Guttman rule. This rule, which is commonly termed *eigenvalues greater than 1* is the most commonly used method. According to this rule, one computes the eigenvalues of the unreduced dispersion matrix (H) and the number of eigenvalues that exceed the average (of the eigenvalues) determines the number of factors. For a correlation matrix, the average eigenvalue is 1; hence the term *eigenvalues greater than 1* has been commonly used in the literature. By the Kaiser-Guttman rule, the number of factors is also determined to be 3.

### **b. Dynamic factor models**

*Dynamic factor models* are based on the state-space representation such that factors are considered as unobserved variables in the measurement equation. Assume  $H_t$  is a  $(N \times 1)$  vector containing CPI sub-aggregates and  $F_t$  is the matrix of factors extracted from  $H_t$ . A *dynamic factor model* can be presented by equations (2) and (3),

$$H_t = \Lambda F_t + e_t \quad (2)$$

$$F_t = \sum_{l=1}^L B_l F_{t-l} + v_t \quad (3)$$

Where  $\Lambda$  is a  $(N \times K)$  factor loading matrix,  $e_t$  is a  $(N \times 1)$  vector of errors following a multivariate normal distribution with zero mean and  $(N \times N)$  covariance matrix,  $B_l$  is a  $(K \times K)$  matrix of the autoregressive and cross-regressive coefficients, and  $v_t$  is a  $(K \times 1)$  vector of random shocks following a multivariate normal distribution with zero mean and  $(K \times K)$  covariance matrix. In a *dynamic factor model*, the common factors have an autoregressive structure and a possible cross-regressive structure. Although the factors at any given time have no direct impact on future values of observed variables, they have an indirect impact as they influence future values of the factors, which, in turn, influence the observed variables concurrently. As before, the number of factors is estimated to be 3 based on the Kaiser-Guttman rule, and the *dynamic factor model* is estimated using MLE.

Figure 2 shows the time series of the factors of CPI sub-aggregates derived by the static and dynamic factor models. As the figure shows, their trends are very similar, but their volatilities are very different; the factor derived by the static model is much more volatile than the one derived by the dynamic model.

**Figure 2: The first factors extracted from CPI sub-aggregates by the static and dynamic factor models**

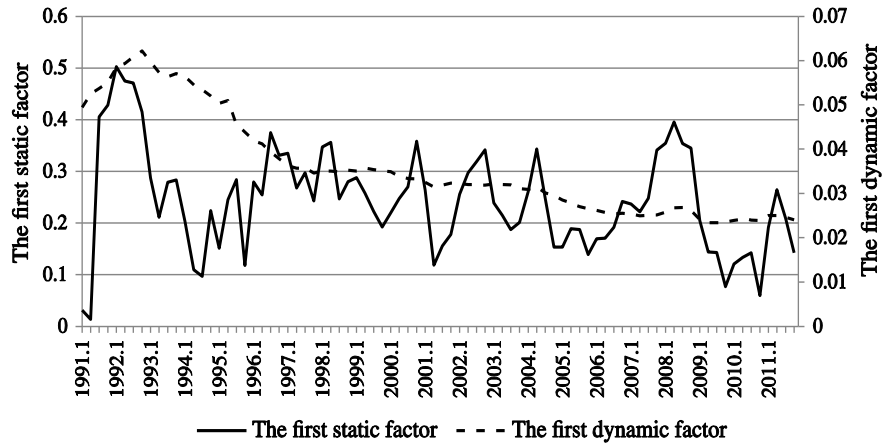


Table 1 shows correlations between the inflation rate, and the factors extracted from CPI sub-aggregates by the static and dynamic models. Correlations between the inflation rate and all static and the first two dynamic factors are positive, but correlation between the inflation rate and the third dynamic factor is negative. The inflation rate has highest correlation with the third factor of the static factors (0.66) and with the first factor of the dynamic factors (0.52).

**Table1: Correlations between the inflation rate, static and dynamic factors of CPI sub-aggregates**

	Inflation	$F_1^S$	$F_2^S$	$F_3^S$	$F_1^D$	$F_2^D$	$F_3^D$
Inflation	1						
$F_1^S$	0.2	1					
$F_2^S$	0.26	-0.14	1				
$F_3^S$	0.66	0.35	-0.15	1			
$F_1^D$	0.52	0.47	0.16	0.46	1		
$F_2^D$	0.46	0.57	-0.12	0.66	0.61	1	
$F_3^D$	-0.14	-0.79	0.27	-0.35	-0.48	-0.85	1

*Notes:* Static factors ( $F_1^S$ ,  $F_2^S$ ,  $F_3^S$ ) and dynamic factors ( $F_1^D$ ,  $F_2^D$ ,  $F_3^D$ ) are extracted from CPI sub-aggregates using the method of MLE and Kalman filter respectively (See section 2 for more details). The estimation period is 1990:2-2012:1.

## 2.1. Forecasting Models

ARX(p) model is presented by equation (4),

$$\pi_t = \alpha + \sum_{i=1}^p \phi_i \pi_{t-i} + \sum_{j=1}^n \beta_j F_{j,t} + \varepsilon_t \quad (4)$$

Where  $\pi_t$  is CPI inflation,  $p$  is the number of inflation lags ( $0 \leq p \leq 4$ ),  $n$  is the number of factors (here  $n=3$ ), and  $\phi$  and  $\beta$  are the coefficients of the inflation lags and the factor loadings, respectively. The forecasts of  $\pi_{t+h}$  are produced iteratively as:

$$\hat{\pi}_{t+h} = \hat{\alpha} + \sum_{i=1}^p \hat{\phi}_i \hat{\pi}_{t-i+h} + \sum_{j=1}^n \hat{\beta}_j F_{j,t} \quad (5)$$

Dynamic ARX(p) is constructed by adding lags of the factors to equation (4):

$$\pi_t = \alpha + \sum_{i=1}^p \phi_i \pi_{t-i} + \sum_{j=1}^n \sum_{m=0}^v \beta_{jm} F_{j,t-m} + \varepsilon_t \quad (6)$$

Where  $v$  is the number of lags of the factors, and  $0 \leq v \leq 4$ .

The forecasts of  $\pi_{t+h}$  are generated iteratively based on (6) as:

$$\hat{\pi}_{t+h} = \hat{\alpha} + \sum_{i=1}^p \hat{\phi}_i \hat{\pi}_{t-i+h} + \sum_{j=1}^n \sum_{m=0}^v \hat{\beta}_{jm} F_{j,t-m} \quad (7)$$

ARMAX(p, q) and Dynamic ARMAX(p, q) models are constructed by adding moving average (MA) terms to the ARX(P) and Dynamic ARX(P), respectively. The number of the lags of MA terms can take each integer number between 1 and 4.

FAVAR model can be presented by equation (8),

$$\begin{bmatrix} F_t \\ \pi_t \end{bmatrix} = \Theta(L) \begin{bmatrix} F_{t-1} \\ \pi_{t-1} \end{bmatrix} + v_t \quad (8)$$

Where  $F_t$  contains the factors extracted from CPI sub-aggregates, and  $\Theta(L)$  is a conformable lag polynomial of finite order  $d$ . The error term  $v_t$  has zero mean with covariance matrix  $Q$ . After estimating factors,  $\hat{F}_t$ , we estimate equation (8) and forecast  $h$ -step ahead inflation iteratively following equation (9):

$$\begin{bmatrix} \hat{F}_{t+h} \\ \hat{\pi}_{t+h} \end{bmatrix} = \hat{\Theta}(L) \begin{bmatrix} \hat{F}_{t-1+h} \\ \hat{\pi}_{t-1+h} \end{bmatrix} \quad (9)$$

In all ARMAX and FAVAR models the optimal number of the lags of the variables and also the lags of moving average terms are selected based on the Schwartz criterion.

Our benchmarks in this paper are Auto Regressive (AR) and single exponential smoothing (SES) models. Quite often such simple models are found hard to beat compared to large complex models. The

form  
is as below,

of

AR

$$\pi_{t+1} = \alpha + \sum_{i=1}^p \beta_i \pi_{t+1-i} + \varepsilon_t$$

(10)

The forecasts of  $\pi_{t+h}$  are constructed recursively as:

$$\hat{\pi}_{t+h|t} = \hat{\alpha} + \sum_{i=1}^p \hat{\beta}_i \hat{\pi}_{t+h-i}$$

(11)

where  $\hat{\pi}_{j|t} = \pi_j$  for  $j \leq t$ ,  
and the specific formula for simple exponential smoothing is:

$$S_t = \alpha \pi_{t-1} + (1 - \alpha) S_{t-1}$$

(12)

There are many ways of setting the initial value of  $S_t$ , such as setting it to  $\pi_1$ . Another possibility, that we use it, would be to average the first four observation.<sup>1</sup>

The forecasts of  $\pi_{t+h}$  are produced as:

$$\hat{\pi}_{t+h} = S_{t+1} = \alpha \pi_t + (1 - \alpha) S_t$$

(13)

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<sup>1</sup> For each of the alpha value between 0.01 and 0.99, we produce out of sample forecasting, then, the alpha corresponding to the lowest RMSFE is selected.

### 3. Data and Results

Our sample quarterly data spans from 1990:2 to 2012:1 including CPI and its 12 sub-aggregates.<sup>2</sup> All forecasts are computed using out-of-sample forecasting framework. For this, we split the sample into two parts: estimation period (1990:2 until 2008:2), and forecast evaluation period (2008:3 until 2012:1). When estimating models we use expanding windows, i.e. first we estimate models using data up to 2008:2 and produce 1 to 4 step ahead forecasts for 2008:3-2012:1. Then we expand the window by one observation and estimate the models using data up to 2008:3 and produce forecasts for 2008:4-2012:1. This procedure is continued until we reach 2011:4.

The data is used in logarithmic, X-12 seasonally adjusted form. Diebold and Kilian (2000) show that when a model includes variables in their stationary form, it generates more accurate forecasts compared to the alternative that it includes variables in their I(1) form. We, therefore, carry out the Augmented Dickey-Fuller (ADF) test for logarithmic CPI and its components, and then, for modeling and forecasting, we use the first difference of those series found to be I(1) by the ADF test.

We showed that the optimum number of factors is 3. However, in order to examine the sensitivity of the results to the number of factors, out-of-sample forecasts for each factor model have been produced when the number of factors is considered to be 1, 2 or 3. For each class of factor models, we report the results for the models with least RMSFEs.

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<sup>2</sup>. Sub-aggregates of the Iranian CPI basket are Food and Beverage; Tobacco; Clothing and Footwear; Housing, Water, Electricity, Gas and other Fuels; Furnishings, Household Equipment and Routine-Household Maintenance; Health; Transport; Communication; Recreation and Culture; Education; Restaurant and Hotels; Miscellaneous Goods and Services.



**Table 2: Relative RMSFEs**

	Forecast Horizon							
	$h=1$		$h=2$		$h=3$		$h=4$	
<i>Factor Models</i>	AR	SES	AR	SES	AR	SES	AR	SES
	0.895	0.885	0.928	1.037	0.942	1.095	1.060	1.233
ARX	(0.21)	(0.41)	(0.01**)	(0.86)	(0.57)	(0.63)	(0.52)	(0.44)
	$(F_1^S)$		$(F_1^S)$		$(F_1^S, F_2^S)$		$(F_1^S)$	
	0.974	0.964	0.895	1.000	1.018	1.183	0.998	1.160
Dynamic ARX	(0.37)	(0.87)	(0.21)	(1.00)	(0.46)	(0.47)	(0.98)	(0.38)
	$(F_1^S)$		$(F_1^S, F_2^S, F_3^S)$		$(F_1^S, F_2^S)$		$(F_1^S, F_2^S)$	
	0.880	0.871	0.940	1.050	0.944	1.097	1.044	1.214
ARMAX	(0.28)	(0.31)	(0.03**)	(0.79)	(0.58)	(0.62)	(0.69)	(0.50)
	$(F_1^S, F_2^S, F_3^S)$		$(F_1^S, F_2^S, F_3^S)$		$(F_1^S, F_2^S)$		$(F_1^S)$	
	1.061	1.050	1.045	1.167	1.035	1.203	0.998	1.160
Dynamic ARMAX	(0.64)	(0.79)	(0.52)	(0.47)	(0.42)	(0.42)	(0.95)	(0.44)
	$(F_1^S, F_2^S, F_3^S)$		$(F_1^S, F_2^S, F_3^S)$		$(F_1^S)$		$(F_1^S)$	
	0.867	0.858	0.882	0.985	0.893	1.038	0.908	1.055
FAVAR	(0.22)	(0.22)	(0.44)	(0.87)	(0.54)	(0.72)	(0.67)	(0.49)
	$(F_1^D, F_2^D, F_3^D)$		$(F_1^D, F_2^D, F_3^D)$		$(F_1^D, F_2^D)$		$(F_1^D, F_2^D)$	

**Notes:** This table reports the Relative RMSFEs (the RMSFEs of factor models relative to the RMSFEs of two benchmark models) for  $h$ -steps ahead forecasts. For each class of factor models, the results for the models with the least RMSFE are reported. The second row of this table for each class of factor models is the factor(s) name(s) which produce the most accurate forecast. The third row for each model represents the P-value of Modified Diebold Mariano test. 5 percent significance level is denoted by two asterisks. The estimation period is 1990:2-2008:2 and the forecast evaluation period is 2008:3-2012:1.

In Table 2, the relative RMSFEs, calculated as the RMSFE of a factor model divided by the RMSFEs of the AR and SES models, and the results of the Modified Diebold-Mariano test<sup>3</sup> are shown. The results show that in most cases the performance of the factor models is better than the AR model, but is not better than the SES model, based on relative RMSFEs. For all horizons, the FAVAR model is the most accurate one. Based on Diebold Mariano test, only for the horizon of two-step ahead, the performance of the ARX and ARMAX models are significantly better than the AR model (at 5% significance level). In the horizons of one, three and four-step ahead, the performance of factor models are not significantly better

<sup>3</sup>. The Modified Diebold-Mariano test examines if model A significantly outperforms model B in terms of forecast accuracy. For more information, see Harvey *et al* (1997).

than AR model. Also, in all horizons, there is no significant difference between the forecast accuracy of factor models and SES benchmark model.

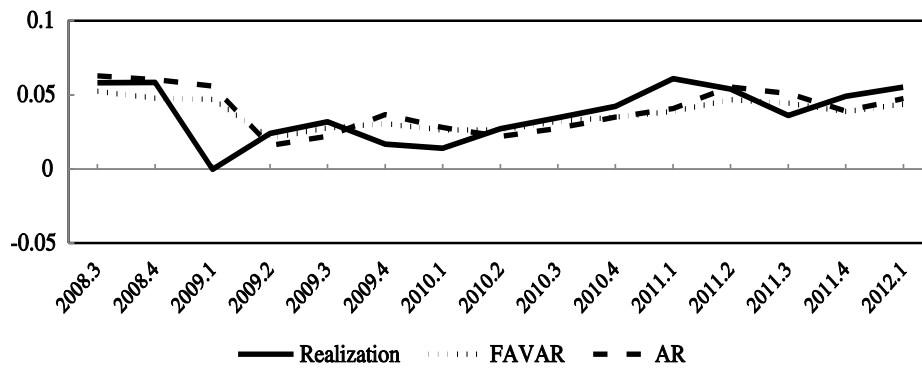
Figure 3 shows the performance of the most accurate factor model (FAVAR) at each forecast horizon. Also, in each forecast horizon, the best benchmark model is shown.

#### 4. Conclusion

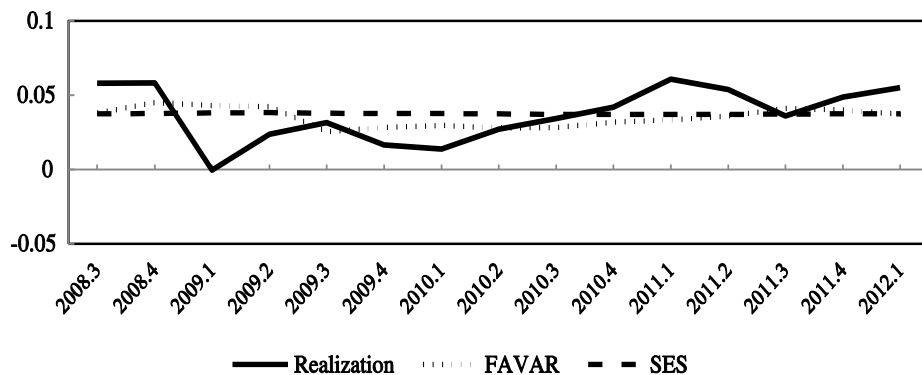
In this paper, we examined whether incorporating the information of CPI sub-aggregates into forecasting models increases the accuracy of forecasts of inflation. The results show that, in general, the factor models, where the factors are extracted from CPI sub-aggregates, outperform the AR model, as one of the benchmark models. Therefore, we can conclude that the information of CPI sub-aggregates has value-added in improving the accuracy of forecasts of inflation.

**Figure 3: Out of sample forecasts by the factor models**

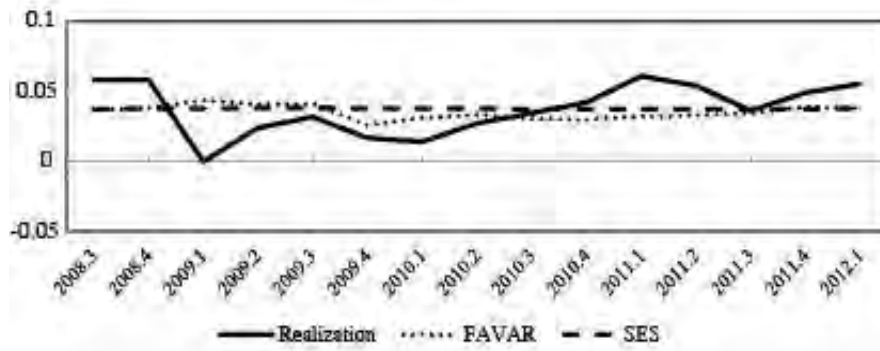
a. One-quarter ahead horizon



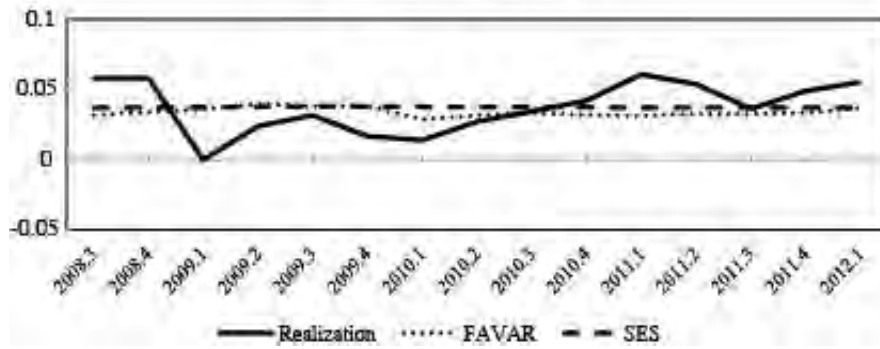
b. Two-quarter ahead horizon



c. Three-quarter ahead horizon



d. Four-quarter ahead horizon



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