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Parametric Estimates of High Frequency Market Microstructure Noise as an Unsystematic Risk

Seifoddini[*], Jalal; Rahnamay Roodposhti[**], Fraydoon and Nikoomaram[***], Hashem

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Abstract

Noise is essential for the existence of a liquid market, and if noise traders are not present in the market, the trade volume will drop severely and an important aspect of the market philosophy will be lost. However, these noise traders bring noise to the market, and the existence of noise in prices indicates a temporary deviation in prices from their fundamental values. In particular, high-frequency prices carry a significant amount of noise that is not eliminated by averaging. If the level of noise in stock prices remains high for a period of time, it can be identified as a risk factor because it indicates that the deviation from fundamental values has been sustained. In this paper, after estimating the microstructure noise in high-frequency prices through a modified parametric approach, using a portfolio switching method, we compared the performance of portfolios having a high level of noise with the performance of portfolios having a lower level of noise and concluded that the risk of the high noise level presents itself as a risk premium in the future return and that asset pricing models which capture the systematic risks cannot capture the noise risk in prices.

Keywords: *Microstructure noise; High frequency data; Quasi-maximum Likelihood Estimation (QMLE); Portfolio switching.*

JEL Classification: *C13, G11, G12*

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[*] Department of Financial Management, Science & Research Branch, Islamic Azad University, Tehran, Iran ; E-mail : jalal.seifoddini@iiau.ac.ir
[**] Department of Financial Management, Science & Research Branch, Islamic Azad University, Tehran, Iran; E-mail : f-rahnamayroudposhti@srbiau.ac.ir
[***] Department of Financial Management, Science & Research Branch, Islamic Azad University, Tehran, Iran; Email: nikoomaram@srbiau.ac.ir

1. Introduction

Market microstructures represent a branch of financial economics that investigates trading and the structure of markets (Harris, 2003). In other words, it investigates the process of financial price formation at the market (Jong & Rindi, 2009). In general, it can be said that market microstructure studies the securities transaction mechanism (Vishwanath & Krishnamurti, 2009). In the present paper, we study noise in prices as an aspect of financial market microstructure. Generally, noise in economies is the opposite of information. This noise is sometimes caused by wrong perceptions and sometimes by false data. Noise can exist anywhere in economy (Black, 1986), but in the financial literature, noise refers mostly to microstructure noise in prices. Noise in prices is a result of structural frictions, such as changes in supply and demand. It also appears due to behavioral factors. In general, any form of temporary deviation in price from its fundamental value is called noise. The role of noise in financial markets is both positive and negative. Fisher Black, in an article published in 1986, was one of the first researchers who investigated how noise affects the financial world. He showed that without noise, no financial market would exist because noise creates liquidity in financial markets. In this area, Hu *et al.* (2013) believe that temporary price deviations involve important information about the level of liquidity in the overall market.

The majority of studies in the field of market microstructure noise have used high-frequency data. High-frequency financial data usually refer to data sampled at a time horizon shorter than a trading day. However, in practice, this definition is not strictly applied and, in some papers, data with daily intervals have been considered high-frequency data. However, the meaning of "high frequency" has changed over the years following the availability of increasingly detailed information on the trading process (Lillo & MiccichÈ, 2010) and because recent advances in information-processing technologies have made it possible to process and analyze financial data at previously unthinkable scales and frequencies. This trend is quite obvious in market microstructure data analysis. Now, unlike before, when average of prices or transaction values over a time period were used in low-frequency analysis, the details of all the transactions can be accessible by researchers depending on the market under study. This has eliminated the opportunity to take advantage of averages instead of all the data. Statistically, it is clear that data averaging reduces the impact of outlier data in the analysis, so the noise in prices would be mitigated, but when all data are used, outlier data are incorporated into analysis, which can affect the results; in other words, the results would be noisy. Hence, in this paper, we try to determine the magnitude of the noise in the volatility estimates from high-frequency data and separate it from the price process.

According to some researchers, the existence of noise in a market makes the market not completely efficient. Fisher Black (1986) is the first researcher to introduce this hypothesis. He believes that noise causes the market to be inefficient. Noise-based trades make prices deviate from their fundamental values. Therefore, as the amount of noise-based trades increases, the profitability of information-based trades will increase; however this only happens because prices have more noise. Other researchers such as De Long *et al.* (1990) also argue that noise trading can lead to a large divergence between market prices and fundamental values. In contrast, some researchers, such as Friedman (1953), Fama (1965) and Benos (1998), believe that the existence of noise traders does not necessarily impact price efficiency; rather, with the assumption that the nature of noise is a temporary deviation and not a permanent one, market efficiency increases with increased market liquidity. They note that noise traders are met in the market by rational arbitrageurs who trade against them and, in the process, drive prices close to fundamental values (De Long, Shleifer, Summers, & Waldmann, 1990). Of course, both groups of researchers consider the existence of noise traders and the noise that can be caused by the activities of this group of traders to be the vital condition of a liquid market. If there are only information traders present in the market because either side of a deal believes that the other side also trades on information, they are worried that the other side's information may be more accurate than their own; for this reason, the other side has taken an opposite position and, consequently, they are reluctant to make the deal. In other words, the existence of traders who trade for reasons other than information provides the required diversity to the market. Thus, noise-based trades are essential for market liquidity (Morawski, 2008). As a result, in this regard, there are two hypotheses concerning noise in the market: “The efficient market hypothesis” and “the noisy market hypothesis” (Bodie, Kane, & Marcus, 2009). Therefore, the issue that this research considers is whether market microstructure noise can be explained by asset pricing models such as the Capital Asset Pricing Model (CAPM)[1], which is based on the efficient market hypothesis, or its

1. CAPM: Capital Asset Pricing Model

existence can generate excess returns, which indicates that the market is not efficient.

So far, researchers who use high-frequency data have often looked for ways to eliminate these noises in their studies and accordingly used methods such as simulation and data filtering. However, in order to examine our hypothesis in this study, instead of eliminating the market microstructure noise, we look for a way to disentangle high-frequency observations on a fundamental component and a microstructure noise component of the stocks' transaction prices. Therefore, based on the research purposes and questions, after estimating the market microstructure noise in prices, the main hypothesis that will be examined is "whether the high-level of noise in high-frequency price data is priced as a risk premium in stock returns and whether this return can be explained by efficient market asset pricing models".

After estimating noise, we ask whether a high level of noise in prices, as a risk factor, is priced in the market, that is, stocks that co-vary with our high-frequency measure of noise tend to be compensated in the form of higher returns. We examine this question through a portfolio switching approach. In the financial field, switching can occur at two levels: The securities and the portfolio level. Switching, at the securities level, means closing a position on a security and taking a position on a more promising security. Switching at the portfolio level, usually in the family of funds, refers to the transition of an investment from one portfolio to another; in general, the transition from one portfolio to another is called portfolio switching. This may happen in one or more markets. For example, Gooptu (1994) focuses on the issue of the possibility of portfolio switching between emerging markets and considers it one of the risks of emerging markets, and Piasecki (2004) focuses on the executive costs of transition from one portfolio to another in the process of portfolio switching. Grant (1978) focuses on the issue in market timing and portfolio management of what is the number of optimal or ideal times for portfolio switching. We use portfolio switching approach to construct portfolios based on the level of noise in stock prices, and at the end of each month, we switch to the portfolio with the highest level of noise.

Among the studies conducted in Iranian capital market, Abbasian and Farzanegan (2012), and Abbasian et al. (2016) study the effect of noise traders' behavior on Tehran Stock Exchange bubbles using monthly data. However, Seifoddini *et al.* (2016) are the first researchers who estimate noise and true realized volatility of high-frequency price series in Iranian capital market.

2. Methodology

There are two approaches for estimating noise. Studies performed in quote-driven markets usually use market makers quotes. They argue that it is common practice in the realized variance literature to use midpoints of bid-ask quotes as measures of the true prices. While these measures are affected by residual noise, they are generally less noisy measures of the efficient prices than are transaction prices because they do not suffer from bid-ask bounce effects. Thus, these studies use midpoints of bid-ask quotes to measure prices, and they generate models of mid-quote determination based on efficient price and residual microstructure noise [see, e.g., Bandi & Russell (2006) and (2008), Mancino & Sanfelici (2008), Griffin & Oomen (2011)]. On the other hand, studies performed in order-driven markets use transaction prices. In studies performed to estimate noise by using transactions prices, two types of estimators have been used. The first one is a parametric estimator, and the second one is a non-parametric estimator (Ait-Sahalia & Xiu, 2012).

The Maximum Likelihood Estimation (MLE) is the parametric estimator provided by Ait-Sahalia *et al*. (2005), and the non-parametric estimator is called the Two Scales Realized Volatility (TSRV)[1], which is provided by Zhang *et al*. (2005). To determine which one is a better estimator of noise in stock prices, Ait-Sahalia & Yu (2009) performed a Monte Carlo simulation. Based on their investigations, in all cases, the MLE and TSRV estimators of noise are robust to various types of departures from their model's basic assumptions under a wide range of simulation design values, including properties of the volatility and the sampling mechanism. However, TSRV is more sensitive to low sampling frequency, because the rate of convergence of TSRV is $n^{-1/6}$ and it is lower than the rate of convergence of MLE which is $n^{-1/4}$. Hence, because these types of situations will occur fairly often in large samples, MLE could be a more efficient estimator of noise. Other researchers, such as Doman (2010), also used the MLE method provided by Ait-Sahalia and Yu (2009) to estimate noise in transaction prices. Alongside these two researchers, Xiu (2010) stated that MLEs are robust estimators for random volatilities based on his studies. They are also robust and powerful for

1. TSRV: Two Scale Realized volatility.

sampling with random intervals and non-Gaussian market microstructure noises, and because of these misspecifications in the model we better call it Quasi-Maximum Likelihood Estimation (QMLE) [1]. QMLE consists of maximizing an assumed likelihood function. This likelihood function does not necessarily correspond to the true likelihood function. QMLEs are robust with respect to incorrect distributional assumptions. One advantage of this approach is that the true underlying dynamics of the error process need not be known (Schmolck, 2012). Therefore, in the present paper, we use QMLE to estimate noise. The authors also studied the estimation of noise in high frequency price series through TSRV approach in another paper (Seifoddini, Roodposhti, & Nikoomaram, 2016), and we will compare those results with the results of the present paper to provide a more comprehensive interpretation of our results.

To estimate microstructure noise in high-frequency prices, using the QMLE method, we begin with the log prices as follows:

$$X_t = ln(P_t) \tag{1}$$

where *t* represents time and lag returns are calculated as follows:

$$R_{t,h} = X_t - X_{t-h} \tag{2}$$

Then, the realized volatility is defined as follows:

$$RV_t(h) = \sum_{j=1}^{1/h} \left(R_{t-1+jh,h}\right)^2 \tag{3}$$

where *h* represents the time interval between two subsequent observations.

The volatility of a financial instrument in a *t* time period σ_t^2, is defined as its conditional variance of its return given the set of information Ω_{t-1}accessible at *t-1*, or, mathematically speaking:

$$\sigma_t^2 = E\left(\left(R_t - E(R_t|\Omega_{t-1})\right)^2 \middle| \Omega_{t-1}\right) \tag{4}$$

1. QMLE: Quasi-Maximum Likelihood Estimation

Therefore, the volatility is an unobservable variable, and the realized volatility is a possible estimator for it.

In the following, we assume that the process of log prices follows the Itō process (Itō, 2006):

$$dX_t = \mu(X_t;\theta)dt + \sigma dW_t \tag{5}$$

where $X_0=0$, W_t represents a Brownian motion, $\mu(.,.)$ is a drift function, θ represents a drift parameter and σ is an instantaneous volatility or diffusion coefficient, where σ>0.

In such a framework, an ideal ex post volatility estimator of σ_t^2 is the integrated volatility (IV):

$$IV(t) = \int_{t-1}^{t} \sigma^2(u)du \tag{6}$$

Now, based on the quadratic variation theory:

$$RV_t(h) \rightarrow \int_{t-1}^{t} \sigma^2(u)du\,, if\ h \rightarrow 0 \tag{7}$$

Therefore, in the absence of noise, it is possible to use realized volatility (RV) as a consistent estimator of σ_t^2. In fact, we want to estimate volatilities σ_t^2 based on discrete observations obtained in moments $0, \Delta, \ldots. n\Delta = T$. For this purpose, first we must separate noise from real prices and then estimate the true RV of prices (Doman, 2010).

Here, it is possible to consider σ as constant without losing the model efficiency. Additionally, in the high frequency context, the drift component is mathematically negligible. This is validated empirically including a drift which actually deteriorates the performance of variance estimates from high-frequency data because the drift is estimated with a large standard error. Therefore, we simplify the analysis one step further by setting μ=0. Ait-Sahalia *et al.* (2005) showed that the removal of these conditions do not eventually change the results. Xiu (2010) also came up with the same results, although, according to Xiu, the correct model specification features stochastic volatility. However, this model is intentionally miss-specified to be one of constant volatilities. Under this assumption, we perform Quasi-Maximum Likelihood Estimation and analyze the estimator, which is essentially the same as the MLE in Ait-Sahalia *et al.* (2005). Therefore, we have:

$$dX_t = \sigma dW_t \tag{8}$$

We assume that our observations are obtained in equal time intervals Δ. Therefore, the parameter σ^2 is estimated at time T and based on $N+1$ observations at $\tau_0=0$, $\tau_1=\Delta, \dots, \tau_N=N\Delta=T$. In this case, QMLE of σ^2 will be equal to RV.

Now, to include noise in our model, we assume that instead of the observation of the process X at τi times, we observe an erroneous value Y as follows:

$$Y_{\tau i} = X_{\tau i} + \varepsilon_{\tau i} \tag{9}$$

Where the $\varepsilon_{\tau i}$ are i.i.d noise with mean zero, and are independent from the W process. Therefore, our parameter of interest is their variance that we denote it by α^2 and our goal is to estimate it and separate it from the true volatility.

We consider X as the true log price and Y as the observed log price. The Brownian motion W is the process representing the arrival of new information to the market, which, in this idealized setting, is immediately impounded in X.

Equation (9) is the simplest form of the market microstructure model.

Now, if we set $\alpha=0$, by considering Y the log price, the return of the observed prices will be equal to the first-difference of the observed log prices as follows:

$$R_i = Y_{\tau_i} - Y_{\tau_{i-1}} = \sigma\left(W_{\tau_i} - W_{\tau_{i-1}}\right),\ \ i = 1, \dots, N \tag{10}$$

Therefore, the R_i are of i.i.d type and with $N(0,\sigma^2\Delta)$ distribution, and their quasi-likelihood function is as:

$$l(\sigma^2) = -N ln\,(2\pi\sigma^2\Delta)/2 - 2\sigma^2\Delta^{-1}R'R \tag{11}$$

where $R=(R_1, \dots .R_n)'$. In this case, the QMLE for σ^2 coincides with the discrete approximation to the quadratic variation of the process:

$$\hat{\sigma}^2 = \frac{1}{T}\sum_{i=1}^{N} R_i^2 \tag{12}$$

Now, if the microstructural noise ε with the mentioned characteristics is present, the true structure of the observed log returns R_i is given by an MA(1) process because

$$R_i = Y_{\tau_i} - Y_{\tau_{i-1}} = X_{\tau_i} - X_{\tau_{i-1}} + \varepsilon_{\tau_i} - \varepsilon_{\tau_{i-1}} = \sigma\left(W_{\tau_i} - W_{\tau_{i-1}}\right) + \varepsilon_{\tau_i} - \varepsilon_{\tau_{i-1}} \equiv u_i + \eta u_{i-1} \tag{13}$$

where $u_i s$ are uncorrelated with mean zero and variance γ^2. The relationship between the main parameters (σ^2, α^2) based on the two equivalent processes above and with the use of MA(1) characteristics will be as follows:

$$Var(R_i) = Var\left(\sigma\left(W_{\tau_i} - W_{\tau_{i-1}}\right) + \varepsilon_{\tau_i} - \varepsilon_{\tau_{i-1}}\right) = Var\left(\sigma\left(W_{\tau_i} - W_{\tau_{i-1}}\right)\right) + Var\left(\varepsilon_{\tau_i} - \varepsilon_{\tau_{i-1}}\right) \tag{14}$$

where

$$Var\left(\varepsilon_{\tau_i} - \varepsilon_{\tau_{i-1}}\right) = E\left[\left(\varepsilon_{\tau_i} - \varepsilon_{\tau_{i-1}} - E\left(\varepsilon_{\tau_i} - \varepsilon_{\tau_{i-1}}\right)\right)^2\right] = E\left[\left(\varepsilon_{\tau_i} - \varepsilon_{\tau_{i-1}}\right)^2\right] = E\left(\varepsilon_{\tau_i}^2\right) + E\left(\varepsilon_{\tau_{i-1}}^2\right) - 2E\left(\varepsilon_{\tau_i}\right)E\left(\varepsilon_{\tau_{i-1}}\right) = \alpha^2 + \alpha^2 + 0 = 2\alpha^2 \tag{15}$$

Therefore, we have

$$\gamma^2(1 + \eta^2) = Var(R_i) = \sigma^2\Delta + 2\alpha^2 \tag{16}$$

Moreover,

$$Cov(R_i, R_{i-1}) = E\left[\left\{\sigma\left(W_{\tau_i} - W_{\tau_{i-1}}\right) + \varepsilon_{\tau_i} - \varepsilon_{\tau_{i-1}} - E\left(\sigma\left(W_{\tau_i} - W_{\tau_{i-1}}\right) + \varepsilon_{\tau_i} - \varepsilon_{\tau_{i-1}}\right)\right\} \times \left\{\sigma\left(W_{\tau_{i-1}} - W_{\tau_{i-2}}\right) + \varepsilon_{\tau_{i-1}} - \varepsilon_{\tau_{i-2}} - E\left(\sigma\left(W_{\tau_{i-1}} - W_{\tau_{i-2}}\right) + \varepsilon_{\tau_{i-1}} - \varepsilon_{\tau_{i-2}}\right)\right\}\right] = E\left[\left(\sigma\left(W_{\tau_i} - W_{\tau_{i-1}}\right) + \varepsilon_{\tau_i} - \varepsilon_{\tau_{i-1}}\right) \times \left(\sigma\left(W_{\tau_{i-1}} - W_{\tau_{i-2}}\right) + \varepsilon_{\tau_{i-1}} - \varepsilon_{\tau_{i-2}}\right)\right] = -E\left(\varepsilon_{\tau_{i-1}}^2\right) = -\alpha^2 \tag{17}$$

and, as a result, we have

$$\gamma^2\eta = Cov(R_i, R_{i-1}) = -\alpha^2 \tag{18}$$

Equivalently, the inverse change of variable is given by:

$$\gamma^2 = \frac{1}{2}\left\{2\alpha^2 + \sigma^2\Delta + \sqrt{\sigma^2\Delta(4\alpha^2 + \sigma^2\Delta)}\right\} \tag{19}$$

$$\eta = \frac{1}{2\alpha^2}\left\{-2\alpha^2 - \sigma^2\Delta + \sqrt{\sigma^2\Delta(4\alpha^2 + \sigma^2\Delta)}\right\} \tag{20}$$

Now, using the data from the observations in the intervals Δ, we estimate the value of σ^2 and α^2 for every time period t, which we consider as equal to one day, using the QMLE. The quasi-likelihood function for the vector R of observed log returns as a function of the transformed parameters (γ^2, η) is given by:

$$l(\eta, \gamma^2) = -\ln det\,(V)/2 - N\,ln\,(2\pi\gamma^2)/2 - (2\gamma^2)^{-1}R'V^{-1}R \tag{21}$$

where

$$V = [v_{ij}]_{i,j=1,\dots,N} = \begin{bmatrix} 1+\eta^2 & \eta & 0 & \dots & 0 \\ \eta & 1+\eta^2 & \eta & \ddots & \vdots \\ 0 & \eta & 1+\eta^2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \eta \\ 0 & \dots & 0 & \eta & 1+\eta^2 \end{bmatrix} \tag{22}$$

and

$$det(V) = \frac{1-\eta^{2N+2}}{1-\eta^2} \tag{23}$$

After estimating the noise via high-frequency price data, we guess that if the value of noise is high in a company for a period of time, then a premium should be considered for it, and consequently, portfolios with higher noise, compared to portfolios formed by stocks having a low level of noise, have a higher return. Additionally, this extra return should not be of systematic type and explainable by the CAPM. To investigate this issue, first, we conduct a test for the effect of noise on return, and if this effect exists, we perform a co-integration test on estimated noises to see if there is a long-term relationship between them. If the co-integration does not exist, then we can conclude that the estimated noise in prices is of an unsystematic nature. After that, we sort the sample companies based on the average noise of their last-month prices and then categorize them into portfolios sorted based on high to low noise. Next, we calculate the return of the mentioned portfolios in the next month. We repeat this action at the beginning of each month. We calculate the average return for the sorted portfolios and investigate whether, by switching from low-noise portfolios to higher-noise portfolios, the return level monotonically increases or not. In order to make sure that the portfolio return was not caused by systematic factors and that we can consider noise as its cause, instead of the return of each portfolio, we use the Treynor ratio of each portfolio. The method for calculation of the Treynor ratio is as follows (Kevin, 2015):

$$\boldsymbol{T} = \frac{\boldsymbol{r_{pi} - r_f}}{\boldsymbol{\beta_i}} \quad (24)$$

where, r_{pi} is the monthly return of the *i(th)* portfolio (here the i^{th} quartile), r_f is the monthly risk-free return, and β_i is the beta of the *i(th)* portfolio, which will be calculated from the weighted average of betas of the shares forming the portfolio.

3. The Data

We perform our study on the shares of companies listed on the Tehran Stock Exchange, which is an order-driven market with no specialist or market maker. One of the characteristics of high-frequency data is that as more data is taken into account, the results will be noisier because each brings its noise to the research. At some point, the amount of data included in the analysis will cause such noise in the research that is more than the information they bring with them. The majority of research conducted in the field of financial market macrostructure usually consider a 5-10 year time period to increase the robustness and credibility of the results. However, in studies conducted on financial markets microstructure, using high-frequency data considering a 1-year time period can provide enough data for the results' robustness. For example, Doman (2010) performed his study on a 1-year time period and only on price data for one stock. Ahn & Cheung (1999) used high-frequency data during a 6-month time period. Tissaoui's research also focused on an auction market, and he performed his study using high-frequency price data for a 3-month time period (Tissaoui, 2012). Accordingly, the time period considered in this research was from early 2015 to early 2016. We set the maximum observation frequency to 5 minutes.

Since we need high-frequency data, the main criterion in the selection of stocks was that they should have the highest amount of trade volume and the highest number of trading days because we require stocks whose trade volume is so high that it becomes possible to obtain the required observations at the determined frequency. Additionally, investigating the relationship between noise and stock return requires stocks that have the lowest number of closing days. Therefore, in order to select the sample stocks, we use stocks included in the list of the 50 most active companies provided by the Tehran Stock Exchange. The list of these companies is seasonally announced by the Tehran Stock Exchange, which ranks companies based on trade volume and the number of trading days. In our research, we select companies represented on this list for four subsequent seasons. The same measures were considered by studies that use high-frequency data; for example, Ait-Sahalia & Yu (2009) removed stocks with fewer than 200 daily trades from their sample. The table below provides the standard deviation, minimum and maximum number of trading days, number of trades per day, daily trading volume and daily volume of orders of the sample in the time period under study.

Table 1. General Characteristics of the Study Sample

Stocks' characteristics	Maximum	Minimum	Average	Standard deviation
Trading days	221.00	184.00	201.32	8.41
Number of trades per day	13,128.00	40.00	355.17	4.12
Daily trading volume	226,785,635.00	1,940.00	2,533,232.03	7.07
Daily volume of orders placed in the trading system	98,019.00	215.00	12,300.56	9.02
The average time interval between orders (s)	240.04	00.00	196.03	10.33

Source: Authors'.

4. Discussion

Based on our research methodology, we obtained the price data with a 5-minute frequency and then estimated the true realized volatility and noise level via the QMLE method. The table below shows the maximum, minimum, average and standard deviation of the noise and true realized volatility of the study sample in the desired period.

Table 2. Summary of Findings about Noise, *RV* and Noise-to-Signal Ratio

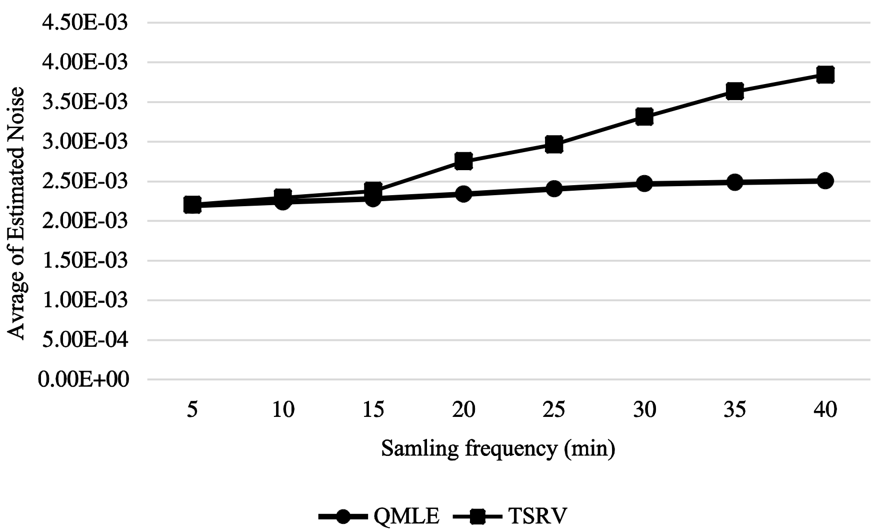
Variable name	**Variable symbol**	**Average**	**Standard deviation**
Noise	$\alpha_{j,t}$	0.00219528	0.0023225
Realized volatility	$\sigma_{j,t}$	0.32097531	0.25493614

Source: Authors'.

The estimated noise and realized volatility are consistent with the noise estimated through TSRV approach in Seifoddini *et al.* (2016), although, the QMLE estimated noise has a lower standard deviation. TSRV is robust to stochastic volatility, but it is more sensitive to low sampling frequency. The bias correction in TSRV relies on the idea that RV computed with all the data. But if the full data sample frequency is low to begin with, as for instance, in the case of a stock sampled every 10 minutes, the bias-correcting may over-correct (Ait-Sahalia & Yu, 2009). Since these types of situations (low sampling frequency) will occur fairly often in stocks listed in Tehran Stock Exchange, the situation argues for privileging QMLE as the baseline estimator of noise in Iran capital market. In the present paper and in our other paper based on TSRV approach (Seifoddini, Roodposhti, & Nikoomaram, 2016) we chose the most liquid stocks listed in Tehran Stock Exchange but if we lower the sampling frequency or choose less liquid stocks, the results of QMLE and TSRV approaches would diverge and, according to the literature review, QMLE estimates would be more accurate than TSRV estimates. Figure (1) highlights the divergence of QMLE and TSRV estimates of noise over different sampling frequencies [see Seifoddini *et al.* (2016) for more explanation about TSRV methodology].

After estimating the microstructure noise in prices, the question remains whether this noise is priced in the market. To answer this question, and to ensure that the existence of the noise affects price return, first we performed the Granger Causality Test. The table below shows the results of the Granger Causality Test regarding the effect of noise on stock returns.

Figure 1: QMLE and TSRV Noise Estimates Using Different Frequencies of Price Series



Source: Authors'.

Table 3. Pairwise Granger Causality Test (Lags: 1)

Null Hypothesis:	Prob.
Preturn does not Granger Cause α	0.4522
α does not Granger Cause *Preturn*	0.000013

Source: Authors'.

As the results of the Granger Causality Test show, it is possible to reject the hypothesis of the lack of an effect of noise on return. Therefore, we can guess that noise affects the stock return.

Now, to test whether noise is a systematic factor in the market, we study the correlation and co-movement of estimated noise in prices of sample stocks. First, we conduct a correlation test between the estimated noises of each pair of the stocks under consideration. The table below shows the results.

Table 4. Correlation Test between the Estimated Noises of Considered Stocks

Correlation test results	Min	Max	Average	STD
Correlation coefficient	-0.803256	0.997948	0.056029	0.263566
Correlation t-statistics	-6.179945	79.4716	1.6581844	9.941064
% positive	61.54%			
% significant	24.18%			

Source: Authors'.

"% positive" indicates that in 61.54% of performed correlations, the correlations were positive, and "% significant" indicates that, based on their t-statistics, only 24.18% of these positive correlations were meaningful. However, in order to come to a more assertive and statistically meaningful conclusion, we check to see if a long-run relationship exists between the estimated noises of stocks under study. In this regard, we use the co-integration test that has been commonly used to study the co-movements and long-run relationship between markets or exchange rates [for example, see (Sireesha , 2013), (Nazlioglu & Soytas, 2012) and (Caporale, Gil-Alana, & Orlando, 2015)]. Now, to perform a co-integration test, we first need to check the stationary conditions of noise series.

Table 5. Unit Root Test for the Estimated Noise Series

Panel unit root test: Summary Exogenous variables: Individual effects Automatic selection of maximum lags Automatic lag length selection based on SIC: 0 to 4 Newey-West automatic bandwidth selection and Bartlett kernel				
	Level		First difference	
Method	Statistic	Prob.	Statistic	Prob.
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t	0.01853	0.5074	-26.3215	0
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	3.36113	0.9996	-25.7143	0
ADF - Fisher Chi-square	10.7824	0.9986	571.694	0
PP - Fisher Chi-square	10.1490	0.9992	603.010	0

Source: Authors'.

As table (5) shows, the null hypothesis is that the variables are non-stationary or the variables have unit root. Due to the low prob. values, at Level form, the null hypothesis could not be rejected. However, at the First Difference, the significance values are below 0.05 for all these values, which implies that the null hypothesis is rejected. Hence, it could be concluded that for all the estimated noise series under study, stationarity has been obtained at the first difference form at a 5% level of significance. This result allows us to perform a co-integration test on estimated noises because if a set of economic series is not stationary, there may exist some linear combination of the variables that exhibit a dynamic equilibrium in the long run (Engle & Granger, 1987). If the variables under study are found to be co-integrated, it will provide statistical evidence of the existence of a long-run relationship. Now, we perform a co-integration test on the estimated noise series. The table below shows the results:

Table 6. Engle-Granger Co-integration Test on the Estimated Noise Series

Null hypothesis: Series are not co-integrated				
Automatic lag specification based on the Schwarz criterion (maxlag=4)				
Maximum and minimum prob. values for dependent variables	tau-statistic	Prob.	z-statistic	Prob.
Min	-6.705349	0.1525	-32.66477	0.0661
Max	-3.577938	0.9707	-17.25225	0.9753
Average	-4.800533	0.6786083	-23.64336	0.6606

Source: Authors'.

Prob. values above 0.05 indicate that there is no long-run relationship between the estimated noises of stocks under study at a 5% level of significance. Hence, according to correlation test and co-integration test results, the estimated microstructure noise is not systematic. Although some studies, such as Hu *et al*. (2013), assert that in the case of an economic shock or a market crash, the noise level increases in the entire market, this is an occasional phenomenon and these situations affect almost every aspect of the market's microstructure and macrostructure in a short period of time.

Now that we are confident about the effect of noise on stock returns and its unsystematic nature, to investigate whether stocks with a higher level of noise can be compensated with higher returns due to a noise risk premium, we consider the return of portfolios categorized based on the noise level. To investigate this matter at the end of each month, we categorize the stocks, based on their average noise in the previous month, into quartiles from a high level of noise to a low level of noise. Then, we calculate the weighted return of each portfolio for the next month. We do this every month and switch to the new categorized portfolios. Then, we compare the return of the portfolio having the highest noise with the quartile portfolio having the lowest noise.

We also categorize and investigate the returns of portfolios based on liquidity measures such as trade volume, number of trades, average volume of each trade, and order volume, as well as variables such as price levels, spread and σ. Before making a conclusion, in order to make sure that the increase in returns was not caused by systematic factors and can be related to noise, instead of the portfolio return, we use the portfolio Treynor ratio.

Table 7. Findings on the Average Monthly Treynor Ratio of Portfolios Sorted Based on Each Measure

	Average of the portfolios' Treynor ratio constructed based on the minimum to the maximum level of each criterion			
Measures	**1 (minimum)**	**2**	**3**	**4 (maximum)**
Noise (α)	0.0057	0.0344	0.0349	0.0527
Daily volume	0.0237	-0.0137	0.0337	0.0507
Daily number of trades	0.0367	0.0047	0.0117	0.0397
Average volume of each trade	-0.0173	0.0397	0.0227	0.0597
Daily orders volume	0.0317	0.0447	0.0037	0.0197
Realized volatility (σ)	-0.0033	0.0307	0.0197	0.0547
Spread	0.0317	0.0447	0.0037	0.0197
Price level	0.0384	0.0407	0.0147	0.0023

Source: Authors'.

As table (7) shows, in portfolios sorted based on noise level, by switching from low-noise portfolios to high-noise portfolios, the portfolio Treynor measure monotonically increases. In contrast, if the risk caused by a high level of noise was explainable by the stocks beta as a measure of systematic risk, we should not have observed this monotonic increase in the portfolios' Treynor measures. We do not see this monotonicity in portfolio returns sorted by other measures. This shows that the noise contains information that other considered measures cannot justify. The results are in line with Seifoddini *et al.* (2016) findings, because both studies were conducted on high frequency price series of most liquid stocks in Tehran Stock Exchange, and as demonstrated in Figure(1), their results may deviate in lower sampling frequencies.

We also conducted this portfolio switching on a quarterly basis, and the results are similar to the monthly portfolio switching.

Table 8. Findings about the Average Quarterly Treynor Ratio of Portfolios Sorted Based on Each Measure

	Average of portfolios' Treynor ratio constructed based on the minimum to the maximum level of each criterion			
Measures	**1(minimum)**	2	**3**	**4(maximum)**
Noise (α)	0.0133	0.0466	0.0733	0.0955
Daily volume	0.0261	-0.0370	0.0809	0.0558
Daily number of trades	0.0849	0.0122	0.0328	0.0801
Average volume of each trade	-0.0569	0.0516	0.0431	0.0955
Daily orders volume	0.0507	0.0626	0.0104	-0.0416
Realized volatility (σ)	0.0023	0.0798	-0.0197	0.0602
Spread	0.0901	0.0492	0.0086	-0.0236
Price level	0.0020	-0.0488	0.0074	0.0048

Source: Authors'.

5. Conclusions

In the present research, we estimated the microstructure noise in prices using high-frequency data and a modified parametric approach. Then, we investigated the hypothesis that the risk caused by the existence of noise in prices, due to sustained deviation in prices from fundamental values, would be priced in the market as a risk premium. Moreover, this kind of risk is not systematic, and asset pricing models based on systematic risks cannot capture the noise risk premium.

We investigated this hypothesis through co-integration and portfolio switching approaches, and based on our findings, we can conclude that if the average noise in the prices of a stock is high for a time period, it can be considered as a risk for the stock and it is compensated by future returns. This research also investigated whether this return can be explained by full systematic risk-based pricing models. Our findings are in line with the findings of Black (1986), De Long *et al.* (1990) and Seifoddini *et al.* (2016) in support of the “the noisy market hypothesis”.

Therefore, based on the results of this research, it is recommended that stock analysts keep in mind that in stocks with a high level of microstructure noise in their prices (which may be due to the presence of noise traders), a premium should be considered when estimating the expected returns. Moreover, as the liquidity of stocks listed in Tehran Stock Exchange is usually low, we suggest that, instead of using TSRV approach explained in Seifoddini *et al.* (2016), investors use QMLE as the main estimator of high-frequency microstructure noise.

References

Abbasian, E., & E. Farzanegan, (2012). "Tehran Stock Exchange Bubbles and Noise Traders Behavior". *Journal of Economic Research (Tahghighat-e-Eghtesadi), 46*(3), pp. 133-153.

Abbasian, E., E. Farzanegan, & E. Nasiroleslami, (2016). "Price Bubble Anomalies in Tehran Stock Exchange: Limits to Arbitrage Approach". *Quarterly Journal of Economic Research and Policies, 23*(76), pp. 75-92.

Ahn, H.-J., & Y.L. Cheung, (1999). "The Intraday Patterns of the Spread and Depth in a Market Without Market Makers: The Stock Exchange of Hong Kong". *Pacific-Basin Finance Journal, 7*(5), pp. 539-556.

Ait-Sahalia, Y., & D. Xiu, (2012). "Likelihood-Based Volatility Estimators in the Presence of Market Microstructure Noise." In L. Bauwens, C. M. Hafner, & S. Laurent, *Handbook of Volatility Models and Their Applications,* p. 348. John Wiley & Sons.

Ait-Sahalia, Y., & J. Yu, (2009). "High Frequency Market Microstructure Noise Estimates and Liquidity Measures". *Annals of Applied Statistics, 3*(1), pp. 422-457.

Ait-Sahalia, Y., P. Mykland, & L. Zhang, (2005). "How Often to Sample a Continuous-time Process in the Presence of Market Microstructure Noise". *Review of Financial Studies, 18*(2), pp. 351-416.

Bandi, F. M., & J.R. Russell, (2006). "Separating Microstructure Noise from Volatility". *Journal of Financial Economics, 79*(3), pp. 655-692.

Bandi, F. M., & J.R. Russell, (2008). "Microstructure Noise, Realized Variance, and Optimal Sampling". *The Review of Economic Studies, 75*(2), pp. 339-369.

Benos, A. V., (1998). "Aggressiveness and Survival of Overconfident Traders". *Journal of Financial Markets, 1*, pp. 353-383.

Black, F. (1986). "Noise". *The Journal of Finance, 41*(3), pp.529-543.

Bodie, Z., A. Kane, & A. Marcus, (2009). *Investments* (8th ed.). mcGraw-Hill/Irwin, pp. 368-369.

Caporale, G. M., L.A. Gil-Alana, & J.C. Orlando, (2015). "Linkages between the US and European Stock Markets: A Fractional Cointegration Approach". *International Journal of Finance & Economics*, *21*(2), pp. 143-153.

De Long, J. B., A. Shleifer, L.H. Summers, & R. J. Waldmann, (1990). "Noise Trader Risk in Financial Markets". *Journal of political Economy,* pp. 703-738.

Doman, M. (2010). "Liquidity and Market Microstructure Noise: Evidence from the Pekao Data". *Dynamic Econometric Models, 10,* pp. 5-14.

Engle, R. F., & C.W. Granger, (1987). "Co-integration and Error Correction: Representation, Estimation, and Testing". *Econometrica: journal of the Econometric Society*, pp. 251-276.

Fama, E. F. (1965). "The Behavior of Stock-market Prices". *The journal of Business, 38*(1), pp. 34-105.

Friedman, M. (1953). *Essays in Ositive Economics.* University of Chicago Press, pp. 159-205.

Gooptu, S. (1994). "Are Portfolio Flows to Emerging Markets Complementary or Competitive?" *Policy Research Working Paper No.1360*, pp. 6-10.

Grant, D. (1978). "Market Timing and Portfolio Management". *Journal of Finance*, pp. 1119-1131.

Griffin, J. E., & R.C. Oomen, (2011). "Covariance Measurement in the Presence of Non-synchronous Trading and Market Microstructure Noise". *Journal of Econometrics*, *160*(1), pp. 58-68.

Harris, L. (2003). *Trading and Exchanges: Market Microstructure for Practitioners.* Oxford University Press, pp. 3-4.

Hu, G. X., J. Pan, & J. Wang, (2013). "Noise as Information for Illiquidity". *The Journal of Finance, 68*(6), pp. 2341-2382.

Itō, K. (2006). *Essentials of Stochastic Processes.* American Mathematical Society, pp. 50-55.

Jong, F. d., & B. Rindi, (2009). *The Microstructure of Financial Markets.* Cambridge University Press, pp. 1-2.

Kevin, S. (2015). *Security Analysis and Portfolio Management* (2nd ed.). PHI Learning Pvt. Ltd, pp.237-238.

Lillo, F., & S. Micciche, (2010). High-Frequency Data. *Encyclopedia of Quantitative Finance*, pp. 1-6.

Mancino, M. E., & S. Sanfelici, (2008). "Robustness of Fourier Estimator of Integrated Volatility in the Presence of Microstructure Noise". *Computational Statistics & data analysis, 52*(6), pp. 2966-2989.

Morawski, J. (2008). *Investment Decisions on Illiquid Assets: A Search Theoretical Approach to Real Estate Liquidity* (1st ed.). Springer Science & Business Media, pp. 20-179.

Nazlioglu, S., & U. Soytas, (2012). "Oil Price, Agricultural Commodity Prices, and the Dollar: A Panel Cointegration and Causality Analysis." *Energy Economics, 34*(4), pp. 1098-1104.

Piasecki, K. (2004). "Optimization Costs of Switching Portfolio as Transportation Problem". *Operations Research and Decisions, 2,* pp. 51-60.

Schmolck, B. (2012). *Omitted Variable Tests and Dynamic Specification: An Application to Demand Homogeneity*. Springer Science & Business Media, pp. 31-32.

Seifoddini, J., F.R. Roodposhti, & H. Nikoomaram, (2016, May). "Microstructure Noise Estimates and Inference Regarding Stock Returns: a Nonparametric Approach." *International Journal of Humanities and Cultural Studies, Special Issue*, pp. 1367-1378.

Sireesha , B. P. (2013). "Co-movements of the Indian Stock Market With Select Emerging Markets". *Journal of Management*, p. 32.

Tissaoui, K. (2012). "The Intraday Pattern of Trading Activity, Return Volatility and Liquidity: Evidence from the Emerging Tunisian Stock Exchange". *International Journal of Economics and Finance, 4*(5), pp. 156-176.

Vishwanath, R., & C. Krishnamurti, (2009). *Investment Management: A Modern Guide to Security Analysis and Stock Selection.* Springer Science & Business Media, pp.13-14.

Xiu, D. (2010). "Quasi-maximum Likelihood Estimation of Volatility with High Frequency Data. *Journal of Econometrics*(159), pp. 235–250.

Zhang, L., P. Mykland, & Y. Ait-sahalia, (2005). "A Tale of Two Time Scales: Determining Integrated Volatility With Noisy High-Frequency Data". *Journal of the American Statistical Association, 100*(472), pp. 1394–1411.