Central Bank Lending, Inflation and Output Dynamics in a Limited Participation Model

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Abstract

This paper seeks to evaluate the effects of expansionary monetary policy by the central bank on inflation, output and employment in a limited-participation model for different parameters value of inter-temporal elasticity of substitution and the Frisch (inverse of the labor supply) elasticity. To this end, a modified cash in advance model in line with Lucas (1990), Christiano (1991) and Fuerst (1992) is utilized to account for liquidity effect and anticipated inflation effect simultaneously. The utility function in this model has a CRRA specification in order to perform the sensitivity analysis of the key variables of the model. A range of parameter values for the economy of Iran is used to show the dependency of the results on the size of the coefficient of relative risk aversion and the Frisch elasticity. The simulation results, given the structure and limitation of our model, show that the injection of liquidity to the banking system is inflationary and is not a reliable policy to increase output. We obtained similar results with different parameter values, except for the risk

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aversion parameter below unity and Frisch elasticity below two. Consistent with prior research, when these parameters are higher, the liquidity effect on output and employment becomes weaker.

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1. Introduction

The Central Bank of I. R. of Iran has conducted expansionary policies through lending to the commercial banks in the recent years. The extra lending capacity generated through central bank lending is argued to boost credit expansion by banking system thus considered by its proponents as a pro-growth policy. In this paper we want to examine the effects of this policy on output and inflation. One way to go about this question is to treat it as a classic type of expansionary monetary policy, the effect of which has widely been examined via the benchmark models of money in a dynamic general equilibrium setting.[1] An alternative approach is to extend these models to include financial intermediation to link bank deposits to credit demand. In this setting, a credit channel is added to the benchmark cash-in-advance models. In this paper, we focus on one form of credit demand, that is, working capital by the firms. This extension provides a more suitable framework to examine the effects of central bank liquidity injection, via lending to commercial banks, on inflation, output and employment in a cash-in-advance framework. Note that the credit market extension, alone, does not change "super-neutrality" property of the standard cash-in-advance models. By setting up a framework where monetary policy matters in the short run because of financial disconnectedness in an economy- "limited participation" models—a variant of cash-in-advance models can generate non-neutrality results. This non-neutrality stems from the intra-temporal efficiency condition in the model.

This paper is based on the modified cash-in-advance model of Lucas (1990), Christiano (1991) and Fuerst (1992) to account for liquidity effect. The modification to standard cash-in-advance model is based on limiting the information set of consumers and the set-up where households cannot continuously revise their consumption and saving plans. Addition of portfolio adjustment costs, along with assumption of a different information structure by households, allows cash-in-advance models to explain certain stylized facts of monetary policy, in particular, the "liquidity effect" phenomenon. By limiting

1. In the context of the above mentioned model and for the Economy of Iran see Jalali-Naini and Naderian (2011) and Shahmoradi et al. (2011).

the information set of one type of agents (households) relative to others in the credit market, “limited participation” models can deliver a real effect, through liquidity effect, in a world of full-information and complete markets, where forward looking, representative rational agents operate.[1] More specifically, since following Lucas (1990), Fuerst (1992), household makes her portfolio decision before she can observe current technology and monetary shocks, the ability to adjust her portfolio is restricted. By introducing a financial intermediary and a portfolio adjustment cost function Christiano and Eichenbaum (1992) showed that a monetary expansion can result in an excess supply of funds a transitory reduction of nominal interest rate (the liquidity effect) in this model.

Assume that the central bank carries out its open market transactions directly with the banking system and injects reserves to the banking system. Households cannot immediately adjust their plans to the altered conditions in the money market. In contrast, the banks are assumed to be in continuous contact with the firms that they keep a lending-borrowing relationship. This modification implies that there is differential sectoral information giving rise to a liquidity effect. Since households have already committed to their plan in the short-run, firms have to absorb any new money injection by the central bank. This liquidity effect will result in a lower nominal interest rate which, in turn, lowers firm's borrowing costs inducing them to borrow more and expand the scale of their activity by employing more workers. This can potentially increase the pace of economic activity if it can dominate the anticipated inflation effect emanating from the money injection shock. Whether or not that the liquidity effect can dominate depends on the relationship the variables of the model and/or the values of its parameters.

The model consists of three distinct representative agents, the consumer, the firm, and the bank. The utility function for consumption and leisure in the above mentioned papers has a logarithmic specification. The log-linearized solution of the model around the steady state is utilized to generate impulse-response

1. Most other DSGE models fail to “reproduce this empirical observation because, in their stylized world of rational representative agents operating with full information in complete markets, higher monetary growth means high inflation in the future. Since agents are interested in real returns, they will demand a nominal return that will equal their desired real return plus the expected rate of inflation...”

effects of an exogenous expansion of base money on the endogenous variables of the model. The utility function has a CRRA specification and a standard labor supply function. As a result, we can measure the sensitivity of the key variables of the model to different values of the inter-temporal elasticity of substitution and the Frisch (inverse of the labor supply) elasticity. We utilize this model to evaluate the impact of expansionary credit-monetary policy by the central bank on inflation, output, and employment. We use a range of parameter values—including those from the economy of Iran—and show that the results depend on the size of the coefficient of relative risk aversion and the Frisch elasticity. We find that, when these parameters are higher the liquidity effect becomes weaker.

2. Consumer Optimization

The set-up is as follows. At the beginning of each period (t), the representative consumer (household) owns the entire stock of money in the economy (M_t) accumulated from labor and capital (interest and dividend) income in the prior period. In the first part of every period consumers disburse their money holding to pay for their purchases from firms, lend to the financial intermediaries which, in turn, lend them to firms. Additional money flows into this economy via injection of money by the central bank into banks which lend them to firms and money for the price level (P_t) is known. The consumer observes the state of the economy but not the shock. At the end of each period money circulates back to the representative consumer in the form of wages and dividends (from firms) and deposited in the bank and subsequently relents by them to firms.[1] There are two other sources of cash payments to consumers. One part comprises the repayment of loans by firms to banks which they borrowed from consumers. The other part consists of bank dividend payment to consumers that originates from repayment of loans by firms to banks financed from the reserves injected by the central bank.

In the standard cash-in-advance models the working assumption is that all agents have full flexibility in responding to a disturbance—because the agents

1. For more details see Christiano (1991).

observe both the state variables and the current monetary and technology shocks. In contrast to the basic CIA models the consumer decides how to divide her money holdings between loans and consumption and how to allocate her time between leisure and labor, the Limited Participation models (Lucas, Christiano 1991 and 1992, Fuerst 1992) assume that the consumer makes her portfolio decisions before she find out the current period value of the shock(s). Moreover, decisions by banks and firms (regarding output, employment and investment) in the model are assumed to be flexible and are made with the knowledge of the current period shock.

$$max\, E_0\left[\sum_{t=0}^{\infty}\beta^t\,[\,\frac{c_t^{1-\sigma}}{1-\sigma}+\Psi\,\frac{(1-H_t)^{1-\eta}}{1-\eta}\,]\right] \quad (1)$$

Where c stands for consumption and 1-H, hours worked. The household faces three constraints. It is assumed that households need to pay cash when purchasing the consumption good produced in the economy (c), i.e. they face a cash-in-advance (CIA) constraint. The representative household could purchase the consumer good with the cash they carry over from the previous period. According to (2) the households can deposit a fraction of their income each period with the financial intermediary if their consumption expenditures fall short of their prior money holdings.

$$c_t = m_t - d_t \quad (2)$$

The second constraint states that the household holds non-negative amounts of real deposits $d_t(=\frac{D_t}{P_t})$ or, in other words, her net position with the bank (deposits – loans) is positive.

$$d_t \geq 0 \quad (3)$$

Third, the household has four sources of money at the beginning of each period. These are labor income in the previous period, $W_{t-1}H_{t-1}$; interest earnings on cash deposited to financial intermediary in the previous period, $R_{t-1}D_{t-1}$; dividend payments from financial intermediaries, B_{t-1}; and dividend payment from firms, F_{t-1}. Each household budget constraint specifies that its uses and sources of money be equated (4):

$$(m_t - d_t) + d_t = R_{H,t-1}\frac{d_{t-1}}{1+\pi_t} + \frac{f_t}{1+\pi_t} + \frac{b_t}{1+\pi_t} + \frac{w_{t-1}}{1+\pi_t}H_{t-1} \qquad (4)$$

The representative household owns the representative bank and the firm, B_t, and F_t is the amount of nominal dividends the household receives from the bank and the firm. Moreover, $R_{H,t}$ stands for the gross nominal interest rate the household observes in the deposit market. The Lagrangian function for the representative household is given by equation (5):

$$\begin{aligned} L = Max\sum_{t=0}^{\infty} E_t(\beta^t [(\frac{c_t^{1-\sigma}}{1-\sigma} + \Psi\frac{(1-H_t)^{1-\eta}}{1-\eta}) + \lambda_t(m_t - d_t - c_t) \\ + \mu_t(R_{H,t-1}\frac{d_{t-1}}{1+\pi_t} + \frac{f_t}{1+\pi_t} + \frac{b_t}{1+\pi_t} + \frac{w_{t-1}}{1+\pi_t}H_{t-1} - m_t)] \end{aligned} \qquad (5)$$

The first order conditions with respect to consumption, leisure, bank deposit, and money holdings are given below:

$$\frac{\partial L}{\partial c_t} = 0 : c_t^{-\sigma} - \lambda_t = 0 \qquad (6\text{-}1)$$

$$\frac{\partial L}{\partial H_t} = 0 : -\Psi(1-H_t)^{1-\eta} + E_t\beta\frac{w_t}{1+\pi_{t+1}}\mu_{t+1} = 0 \qquad (6\text{-}2)$$

$$\frac{\partial L}{\partial d_t} = 0 : -\lambda_t + E_t\beta R_t\mu_{t+1}\frac{1}{1+\pi_{t+1}} = 0 \qquad (6\text{-}3)$$

$$\frac{\partial L}{\partial m_t} = 0 : \quad \lambda_t - \mu_t = 0 \qquad (6\text{-}4)$$

From the equations (6.1-6.4) we obtain the following relationships:

$$c_t^{-\sigma} = \lambda_t \qquad (7)$$

$$\Psi(1-H_t)^{-\eta} = E_t \beta \frac{w_t}{1+\pi_{t+1}} \mu_{t+1} \qquad (8)$$

$$\lambda_t = E_t \beta R_t \frac{\mu_{t+1}}{1+\pi_{t+1}} \qquad (9)$$

$$\lambda_t = \mu_t \qquad (10)$$

From (7), (8) and (10) we obtain the relationship determining the labor supply that relates the marginal disutility of working an extra hour to the marginal utility of consumption:

$$\Psi \frac{(1-H_t)^{-\eta}}{\beta E_t c_{t+1}^{-\sigma}} = \frac{w_t}{1+\pi_{t+1}} \qquad (11)$$

This is the efficiency condition associated with household's decision about how many to work, H_t. Suppose for example, that a household were to increase labor supply by one unit. The utility cost of this is $-u_{l,t} = -\psi(1-H_t)^{-\eta}$. The benefit that the household earns wage, W_t, which can be spent next period on W_t/P_{t+1} units of the economy's consumption good. The discounted value of this benefit to the household is $\beta E_t u_{c,t+1} \frac{W_t}{P_{t+1}} = \beta E_t c_{t+1}^{-\sigma} \frac{W_t}{P_{t+1}}$. If the household's undisturbed plan were, indeed, optimal, as we suppose, then the costs and benefits of the above one unit deviation from the optimal plan must exactly match as in equation (11).

From (7), (9), and (10) we obtain the dynamic supply of saving.

$$E_t\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \frac{\beta R_{H,t}}{1+\pi_{t+1}} \qquad (12)$$

This is the efficiency condition associated with household's decision on how much money to deposit in financial intermediary, D_t. Suppose that the household increases D_t by one dollar. On the cost side, this would decrease consumption spending by one dollar, which would decrease period t real consumption by $1/P_t$, which would decrease utility by $\frac{u_{c,t}}{P_t}$. On the benefit side, the extra dollar invested with the financial intermediary would generate R_t dollars at the end of the period. These can be used to buy R_t/P_{t+1} units of the consumption good next period. The discounted expected value of those goods is $E_t R_t \frac{u_{c,t+1}}{P_{t+1}} = E_t R_t \frac{c_{t+1}^{-\sigma}}{P_{t+1}}$. If the original plan is optimal, as we suppose, then these cost and benefits must be equal. This equation can be used to gain insight into the model's implication for the link between interest rate and inflation and hence, money growth. Based on this equation, a higher rate of (gross nominal) interest in the current period results in less consumption currently, hence, increases the current supply of deposits.

An equation similar to (12) in a limited participation model is:

$$E_{t-1}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = E_{t-1}\left(\frac{\beta R_{H,t}}{1+\pi_{t+1}}\right) \qquad (13)$$

Assume a marginal increase in D. The opportunity cost of this marginal increase in the limited participation (LP) setting is equal to the right hand side of (13). Specification of the conditional expectation indicates that, at the time the individual (household) chooses the magnitude of bank deposit even though she does not know how much is going to be C_t, P_t or L_t because these values depend on the realization of technology and monetary shocks.[1] The benefit of the marginal increase is D_ts $E_{t-1} R_t \beta (\frac{u_{c,t+1}}{P_t})$. Equality of benefits and costs, hence the relationship (13) holds in the limited participation model.

One can express this condition in a different form. By defining $\Lambda_t = R_t E_t \beta \left(\frac{u_{c,t+1}}{P_{t+1}}\right) - (\frac{u_{c,t}}{P_t})$, such that $E_{t-1}\Lambda_t = 0$, and solving for R_t.

$$R_t = \frac{\Lambda_t + \left(\frac{u_{c,t}}{P_t}\right)}{E_t \beta \left(\frac{u_{c,t+1}}{P_{t+1}}\right)} \qquad (14)$$

1. For more details and discussions see Christiano (1991).

The term Λ_t is the liquidity effect (Fuerst 1992). It measures the relative value of money in the consumer goods and in the market for loan. In case $\Lambda_t < 0$, money has a higher value in the goods market, because, households are willing to borrow at a higher rate than R_t given the opportunity to borrow but firms are willing to borrow at R_t . Hence according to Fuerst (1992) when $\Lambda_t < 0$, the loan market is relatively liquid, and in the case $\Lambda_t > 0$, the goods market is relatively liquid. The inclusion of information friction of limited participation in Cash-in-Advance model results in the violation of no-arbitrage condition for money between the goods and loan markets, which was valid in the basic model. The no-arbitrage condition in the CIA model leads to the formation of Fisher relationship that is not held in the Fuerst-Lucas model.

3. Firm Optimization

Since firms are owned by households a natural assumption is that each firm behaves in the best interest of its shareholders. Thus, it is assumed a firm values a dividend dollar in a particular period t by the marginal utility to the households of a dollar at the end of period t. The firms and the financial intermediary are owned by the household, which pay her a stream of nominal dividends discounted by marginal utility of consumption; hence the firm's objective function can be formulated as:

$$max\, E_0\left[\sum_{t=0}^{\infty}\beta^{t+1}\frac{u_{c,t+1}\times F_t}{P_{t+1}}\right]=max\, E_0\left[\sum_{t=0}^{\infty}\beta^{t+1}\frac{f_t}{c_{t+1}^{\sigma}(1+\pi_t)}\right] \quad (15)$$

Notice that since dividend or retained earnings are sent to household at the end of period and consumption is subject to a cash-in-advance constraint; they will be spent one period after the dividend is made. That is why the dividend is discounted by $c_{t+1}^{\sigma}P_{t+1}$ and not $c_t^{\sigma}P_t$. The household values a unit of nominal dividend in terms of the consumption it facilitates during period t+1.

The firm's costs are wages for which the money has to be borrowed from the banks and investment which is financed by retained earnings. The firm chooses productive factors, hence determines its demand for labor (N_t) and (next period) capital stock, (K_{t+1}), also the amount of loans (L_t) and dividend payments (F_t). The firm faces a resource constraint as in (16):

$$f_t = l_t + \left[A_t k_t^{\alpha} N_t^{1-\alpha} - k_{t+1} + (1-\delta)k_t\right] - w_t N_t - l_t R_{F,t} \qquad (16)$$

where $R_{F,t}$ stands for gross nominal interest rate on loan extended to households and (N) stands for the material and labor used for production. In this model we assume that total working capital is equal to the sum of wage bill plus intermediate materials for production. Since firms have to pay initially for the materials used in the production and wages before production begins, elements of working capital (wages and materials) are financed through borrowing. Therefore, at the beginning of the period the firm borrows sufficiently to finance working capital and pays back the loan with interest after production is sold at the end of the period. The amount of loan at the beginning of the period is given below:

$$l_t = w_t N_t \qquad (17)$$

The Lagrange function for the representative firm is

$$L_t = E_0 \left[\sum_{t=0}^{\infty} \begin{matrix} \beta^{t+1}\left(\dfrac{f_t}{c_{t+1}^{\sigma}(1+\pi_t)} + \lambda_t (l_t + (A_t K_t^{\alpha} N_t^{1-\alpha} - k_{t+1} + (1-\delta)k_t) - w_t N_t - l_t R_{F,t} - f_t)\right. \\ + \mu_t (l_t - w_t N_t) \end{matrix} \right] \qquad (18)$$

The first order conditions are given below:

$$\frac{\partial L}{\partial f_t} = 0 : \beta \left[\frac{1}{c_{t+1}^{\sigma}(1+\pi_t)} - \lambda_t \right] = 0 \qquad (19\text{-}1)$$

$$\frac{\partial L}{\partial k_{t+1}} = 0 : -\beta\lambda_t + \beta^2 E_t(\alpha\lambda_{t+1} A_{t+1} k_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + (1-\delta)) = 0 \qquad (19\text{-}2)$$

$$\frac{\partial L}{\partial N_t} = 0 : \beta[(1-\alpha)\lambda_t A_t k_t^{\alpha} N_t^{-\alpha} - \lambda_t W_t - \mu_t W_t] = 0 \qquad (19\text{-}3)$$

$$\frac{\partial L}{\partial l_t} = 0 : \beta(\lambda_t - \lambda_t R_{F,t} + \mu_t) = 0 \qquad (19\text{-}4)$$

From the FOCs we obtain:

$$\lambda_t = \frac{1}{(1+\pi_t)c_{t+1}^{\sigma}} \qquad (20)$$

$$\lambda_t = \beta E_t(\lambda_{t+1}(\alpha A_{t+1} k_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + (1-\delta)) \qquad (21)$$

$$(1-\alpha)\lambda_t A_t k_t^{\alpha} N_t^{-\alpha} = w_t(\lambda_t + \mu_t) \qquad (22)$$

$$R_{F,t} - 1 = \frac{\mu_t}{\lambda_t} \qquad (23)$$

From (20), (21) the condition for trading off consumption over time can be obtained:

$$E_t \frac{1}{(1+\pi_{t+1})c_{t+1}^{\sigma}} = \beta E_t(\frac{1}{(1+\pi_{t+2})c_{t+2}^{\sigma}}(\alpha A_{t+1} k_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + 1 - \delta)) \qquad (24)$$

This equation considers the firm’s decision to invest. Though firm’s control over investment, confront a trade-off between current and future dividends. For example by setting investment at a high level, a firm raises future dividends at the cost of lower current dividends. Now, suppose a firm increases capital for one dollar, on the cost side it reduces the firm dividends that accrues to household and lowers the real consumption of the next period for $1/P_{t+1}$. The utility cost of this reduction will be $\frac{u_{c,t+1}}{P_{t+1}}$ and the discounted expected value is

$$E_t \beta \frac{u_{c,t+1}}{P_{t+1}} = E_t \beta \frac{c_{t+1}^{-\sigma}}{P_{t+1}}.$$

The value of this extra dollar capital formation for the household will be the rise in the dividend and thus real consumption resulted from net marginal productivity of capital amounted to be:

$$\beta^2 E_t(\frac{u_{c,t+2}}{P_{t+2}}(f_k(K_{t+1},N_{t+1})+1-\delta))=\beta^2 E_t(\frac{1}{(1+\pi_{t+2})c_{t+2}^{\sigma}}(\alpha A_{t+1}k_{t+1}^{\alpha-1}N_{t+1}^{1-\alpha}+1-\delta))$$

Using these costs and benefits of a change in capital optimal plan leads to (24). From (22) and (23) demand for labor is derived as in (25)

$$R_{F,t}=\frac{(1-\alpha)A_t k_t^{\alpha}N_t^{-\alpha}}{w_t} \qquad (25)$$

This equation considers the decision of firm to hire hours of labor, H_t. Suppose a firm considers the following change from its optimal employment plan. It borrows one dollar in period t, at a cost of owing R_t at the end of the period. It uses the dollar to hire $1/W_t$ units of labor time, which increases the firm's revenue by $P_t\frac{f_{N,t}}{W_t}=P_t\frac{(1-\alpha)A_t K_t^{\alpha}N_t^{1-\alpha}}{W_t}$. In equilibrium these costs and benefits must cancel. Note that in the above equation $R_{F,t}W_t/P_t$ is equal to the real marginal cost of labor and the other side of the equation (deflated by P) is equal to the marginal product of labor. The inter-temporal effect of the credit condition is to influence the marginal cost of labor. An increase in the gross rate of interest ($R_{F,t}$=1+r) results in an increase in the marginal cost of labor, hence less demand for labor.

4. Bank Optimization

The bank's inflow of money consists of deposits and new money received through liquidity injection by the central bank. It uses these sources to lend to firms. For the financial intermediary the objective is to maximize the present value of the flow of bank dividend for the owners.

$$max\,E_0\left[\sum_{t=0}^{\infty}\beta^{t+1}\frac{b_t}{c_{t+1}^{\sigma}(1+\pi_t)}\right] \qquad (26)$$

The bank maximizes the above function subject to its budget constraint:

$$b_t=d_t+R_{F,t}l_t-R_{H,t}d_t-l_t+x_t \qquad (27)$$

where, B_t stands for Bonds, D_t for Deposits, L_t for Loan, $R_{F,t}$ for Gross nominal interest return on loan, $R_{H,t}$ for gross nominal interest return on deposits, and $x_t = \frac{m_{t+1}}{1+\pi_{t+1}} - m_t$ is the volume of money injected to the banking system by the central bank. The Lagrangian function for the representative bank is given by (28):

$$L = MaxE_0\left[\sum_{t=0}^{\infty}\beta^{t+1}\left(\frac{b_t}{c_{t+1}^{\sigma}(1+\pi_t)} + \lambda_t\left(d_t + R_{f,t}l_t - R_{H,t}d_t - l_t + x_t - b_t\right) + \mu_t(x_t + d_t - l_t)\right)\right] \quad (28)$$

$$\frac{\partial L}{\partial b_t} = 0 : \beta^{t+1}\left[\frac{1}{c_{t+1}^{\sigma}(1+\pi_t)} - \lambda_t\right] = 0 \Rightarrow \lambda_t = \frac{1}{c_{t+1}^{\sigma}(1+\pi_t)} \quad (29\text{-}1)$$

$$\frac{\partial L}{\partial l_t} = 0 : \beta^{t+1}\left[\lambda_t(R_{F,t} - 1) - \mu_t\right] = 0 \Rightarrow R_{F,t} - 1 = \frac{\mu_t}{\lambda_t} \quad (29\text{-}2)$$

$$\frac{\partial L}{\partial d_t} = 0 : \beta^{t+1}\lambda_t(1 - R_{H,t}) + \beta^{t+1}\mu_t = 0 \Rightarrow R_{H,t} - 1 = \frac{\mu_t}{\lambda_t} \quad (29\text{-}3)$$

Using (29.2), (29.3) we get $R_{F,t} = R_{H,t}$ that shows the equality of gross nominal interest return on loan and gross nominal interest return on deposits in equilibrium. It means there is no spread for the financial intermediary in equilibrium. In this model all markets clear at the same time. Thus we have the following market clearing condition for the labor, money, and the goods market, respectively:

$$H_t = N_t \quad ; \quad (30\text{-}1)$$

$$c_t = m_t + x_t \quad ; \quad (30\text{-}2)$$

$$c_t + (k_{t+1} - (1-\delta)k_t) = A_t k_t^{\alpha} N_t^{1-\alpha}, \quad (30\text{-}3)$$

Moreover, in equilibrium gross rate of interest on deposits equals the gross rate of interest on loans.

$$R_{H,t} = R_{F,t} = R_t \quad (30\text{-}4)$$

From the previous relationships derived from FOCs for the household, the firm, and the financial intermediary and market clearing conditions, we can derive the dynamic system (31-42). We have derived two inter-temporal relationships from the FOCs. The first one (equation 31) derived from equation (20) and (21) represents the propagation mechanism emanating from firms' dividend. The profit made by the firms is used to finance capital accumulation and interest payment to the financial intermediary. The second inter-temporal condition shown as (35)-derived from equation (7), (9), (10) - states that the utility loss involved in forgoing one unit of consumption today in the form of deposit must at all times be equal to the discounted utility of one unit of money saving tomorrow. Based on this condition, rate of return on saving in the form of deposit equals equilibrium gross interest rate (R_t) which is determined in the credit market between firm and financial intermediary. It is possible to say that this condition is the Euler function for the credit market. If these two inter-temporal mechanisms are combined, we can derive the unique inter-temporal mechanism the same as Cooley-Hansen (1989) Cash-in-Advance model.

If we combine these two inter-temporal efficiency conditions for the base-case model (in the absence of limited information), we obtain the same inter-temporal condition as in the Cooley-Hansen (1989) model—as we would in a cash-in-advance model. Deposits in this model play the same role as savings in a CIA model. The difference is in the intra-temporal condition in the labor market. Since labor is a cash-good in this model, therefore, the ratio of the utility of leisure to consumption is not equal to the marginal product of labor. Instead the cost of financing the marginal wage-bill is added to the marginal product of labor, resulting in a lower equilibrium employment.

In this model there are two channels for the transmission of money shocks. The first, like Cooley-Hansen is inflation-tax which influences output through lower consumption and higher leisure. The second channel is the influence of inflationary expectations on the nominal interest rate and the consequent reduction in demand for labor, which makes the model non-neutral. The difference with CIA model is thus the addition of a channel to influence. This channel exacerbates the influence of monetary shock on employment—to be more exact, exacerbates the decline in employment. In the LP model the movement in real wage depends on the price elasticity of labor demand and supply. In the Lucas-Fuerst setup with LP, not only intra-temporal mechanism is different from the Cooley-Hansen model but also the inter-temporal mechanism is different due to differences in the distribution of information across agents which violate the Fisher relationship. It should be noted that the primary usefulness of Fuerst-Lucas-Christiano LP model is to examine whether the anticipated inflation effect stemming from money injection dominates the positive demand effect (due to a lower nominal interest rate). These two effects manifest themselves through the supply of labor (via inflation tax on consumption such as in Cooley-Hansen) and the demand for labor due to the liquidity effect: a reduction in the nominal interest rate and lower marginal cost of employment—due to the cash-good nature of labor-service purchases.

$$E_t \frac{1}{(1+\pi_{t+1})c_{t+1}^{\sigma}} = \beta E_t\left(\frac{1}{(1+\pi_{t+2})c_{t+2}^{\sigma}}(\alpha A_{t+1}k_{t+1}^{\alpha-1}N_{t+1}^{1-\alpha}+1-\delta)\right) \tag{31}$$

$$l_t = w_t N_t \tag{32}$$

$$\Psi \frac{(1-H_t)^{-\eta}}{\beta E c_{t+1}^{-\sigma}} = \frac{w_t}{1+\pi_{t+1}} \tag{33}$$

$$R_t = \frac{(1-\alpha)A_t k_t^{\alpha} N_t^{-\alpha}}{w_t} \tag{34}$$

$$E_{t-1}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta E_{t-1}\frac{R_t}{1+\pi_{t+1}} \tag{35}$$

$$c_t + k_{t+1} = A_t k_t^{\alpha} N_t^{1-\alpha} + (1-\delta)k_t \tag{36}$$

$$c_t = m_t - d_t \tag{37}$$

$$l_t = m_{t+1}(1+\pi_{t+1}) - m_t + d_t \tag{38}$$

$$Y_t = A_t k_t^{\alpha} N_t^{1-\alpha} \tag{39}$$

$$m_t = m_{t-1} - \pi_t + u_t \tag{40}$$

$$ln\, A_t = \rho_A \, ln\, A_{t-1} + \varepsilon_t \qquad \varepsilon_t \equiv iid \tag{41}$$

$$u_t = \rho_u u_{t-1} + \upsilon_t \qquad \upsilon_t \equiv iid \tag{42}$$

Equation (31) indicates the trade-off facing the household for moving consumption goods across time; it is the Euler equation in the equity market. Equation (32) denotes the firm's borrowing constraint. It should be noted that the borrowing constraint significantly influences the demand for the working capital—which the firm spends on labor and raw (intermediate) materials. In this paper the firms use borrowed funds to pay for labor input. To simplify the model and reduce the number of equations the firm-agent that supplies raw materials is not explicitly modeled.

Equation (33) is the labor-leisure choice from which the labor supply by the households can be determined. Equation (34) shows the equilibrium interest rate for which the marginal product of labor equals marginal cost of borrowing for that unit of labor. Euler equation in the credit market is stated in equation (35). Aggregate resource constraint is shown in equation (36). Equation (37) is the money market equilibrium condition and equation (38) is the credit market equilibrium condition and equation (39) is the production function. Equation (40) is the equation of motion for the real money. The other two equations are stochastic processes for technology and money growth. The steady-state ratios for the model are:

$$\frac{Y^{ss}}{k^{ss}} = \frac{1}{\alpha}(\beta^{-1} - 1 + \delta) \tag{43}$$

$$\frac{c^{ss}}{k^{ss}} = YK - \delta \tag{44}$$

$$\frac{l^{ss}}{k^{ss}} = \frac{w^{ss} N^{ss}}{k^{ss}} \tag{45}$$

$$\frac{w^{ss}N^{ss}}{Y^{ss}}=(1-\alpha)\frac{1}{R^{ss}} \quad (46)$$

$$\frac{d^{ss}}{m^{ss}}=\frac{\frac{l^{ss}}{c^{ss}}}{1+\frac{l^{ss}}{c^{ss}}} \quad (47)$$

$$R^{ss}=\frac{\Theta}{\beta} \quad (48)$$

$$d^{ss}=l^{ss} \quad (49)$$

As indicated previously, we have two distinct channels for the impact of monetary variables on real variables. The first channel is that of| inflation-tax—which as described by Cooley & Hanson (1989) influences labor supply (N^{ss}). The second channel is through interest rates. An increase in θ increases the growth rate of the money supply and according to (48) increases nominal interest rate in the steady state, which according to (46) results in a reduction in the share of income going to labor in steady state. The presence of these two channels results in non-neutrality in this model. Note that, regardless of working with a base-case Cash-in-advance or an LP model, the steady state values of the system is the same.

Equations 43 to 49 show the ratio of the variables of interest in the steady state. In equation (48) Θ stands for the steady state growth rate of the money supply which is assumed to be equal to the steady state inflation rate. Before we can calibrate and solve the model, we need to transform it so that all variables follow a stationary process around the steady-state. To do so, we linearize the model around the steady-state. Percentage deviation of a variable u around its steady-state value will be denoted by $\tilde{u}_t$; $u_t = u^{ss}(1+\tilde{u}_t)$. The log-linearized system of equations is as follows—tilda over variables imply percent deviation from steady-states.

$$-\sigma\tilde{c}_{t+1}-\tilde{\pi}_{t+1}+\sigma\tilde{c}_{t+2}+\tilde{\pi}_{t+2}=R^{ss}(1-\beta(1-\delta))(\tilde{y}_{t+1}-\tilde{k}_{t+1}) \quad (50)$$

$$\tilde{w}_t=\tilde{l}_t-\tilde{n}_t \quad (51)$$

$$(\eta \frac{n^{ss}}{1-n^{ss}})\tilde{n}_t = -\sigma\tilde{c}_{t+1} + \tilde{w}_t - \tilde{\pi}_{t+1} \qquad (52)$$

$$\tilde{R}_t = \tilde{y}_t - \tilde{l}_t \qquad (53)$$

$$\sigma E_{t-1}(\tilde{c}_{t+1} - \tilde{c}_t) = E_{t-1}(\tilde{R}_t - \tilde{\pi}_{t+1}) \qquad (54)$$

$$(\frac{Y^{ss}}{k^{ss}})\tilde{y}_t = (\frac{c^{ss}}{k^{ss}})\tilde{c}_t + \tilde{k}_{t+1} - (1-\delta)\tilde{k}_t \qquad (55)$$

$$\tilde{c}_t = \tilde{m}_t - \tilde{d}_t \qquad (56)$$

$$\tilde{u}_t + \frac{d^{ss}}{m^{ss}} d_t = \frac{d^{ss}}{m^{ss}} \tilde{l}_t \qquad (57)$$

$$\tilde{Y}_t = \alpha\tilde{k}_t + (1-\alpha)\tilde{n}_t + \tilde{a}_t \qquad (58)$$

$$\tilde{m}_t = \tilde{m}_{t-1} + \tilde{\pi}_t + u_t \qquad (59)$$

$$\tilde{a}_t = \rho_A \tilde{a}_{t-1} + \varepsilon_t \quad \varepsilon_t \equiv iid \qquad (60)$$

$$u_t = \rho_m u_{t-1} + \upsilon_t \quad \upsilon_t \equiv iid \qquad (61)$$

Calibrated parameters are: $\alpha = 0.54$; $\delta = 0.025$; $\beta = 0.985$; σ (inverse of IES) =2.17, and $\Theta^{ss} = 1.0125$. Based on these parameters it is possible to compute the steady-state values and therefore simulate the model for different technology and money growth shocks. In the following, the results of simulation calibrated for Iran are depicted.

5. Impact of a Monetary Shock

We first simulated the effect of a monetary shock on the endogenous variables of the model using the parameters of the US economy used in Christiano (1991) paper to validate our model. Our simulation results with US parameters are consistent with those obtained by Christiano. We also obtained liquidity effect for the US economy in a case with zero monetary shock persistence and complete depreciation. Next, we simulated the model with parameters for the economy of Iran (shown in the previous section) and the impulse-responses of the effect of a once-for-all monetary nominal monetary growth shock (Θ) due to a policy-determined increase in the value of X on the main endogenous variables of the model in the model shown in figure (1).

Output initially rises slightly (due to increase in investment) for two quarters then falls below steady state for an extended period until it reverts to the steady state. The time profile of output in the LP case is somewhat similar to that in a cash-in-advance model. The profile of impulse-responses for real money balances and real loan are similar and show similar time profiles in the LP and CIA models. Consumption in the LP model initially falls for a relatively short period but rises slightly above the steady state level for an extended period until reverting to the steady state. Investment in LP and CIA models both jump initially but quickly revert to the steady state level. The same can be said for real wage dynamics.

There are two opposing forces on the movements of nominal interest rate. One is the effect of anticipated inflation which is a feature of cash-in-advance models with rational expectations. The other one is the presence of the liquidity effect which compensates for the rise in nominal interest rates due to anticipated inflation. Interest rate, in both LP and CIA, rise, indicating the dominance of inflationary expectations, and gradually revert to the steady state. Reversion to the steady state is more gradual in the case of LP model. The observation that there is no initial decline of the nominal interest rate represents the absence of the liquidity effect. As indicated, employment declines and remains below the steady state for an extended period, further signaling the dominance of inflationary expectation effect in response to liquidity injection.

The simulation results show that the introduction of limited participation into the cash-in-advance model does not result in the emergence of a liquidity effect under plausible parameter values for the economy of Iran, even under implausible values as in the case of zero persistence and complete depreciation. In this particular case, we obtain no steady state solution. Given the results, inflationary expectation effect dominates any positive demand effect due to injection of liquidity by the central bank; we find no evidence of liquidity effect in the sense of Christiano.

6. Sensitivity to Parameter Values

In this model the household has one inter-temporal (consumption/ deposit) and one intra-temporal (labor/leisure) choice to make. The representative firm has an inter-temporal (investment/dividend) and intra-temporal (employment/loan) choices to make. One can combine these four choices, to reduce them to two general equilibrium choices—omitting prices, wage and interest rate. In the inter-temporal choice, the crucial parameter is the inter-temporal elasticity of substitution. In the intra-temporal condition, labor supply elasticity is the crucial variable. In this section we examine the sensitivity of model to different values of risk-aversion coefficient and the Frisch elasticity.

Christiano (1991) argues that higher values of the risk aversion coefficient and Frisch elasticity results in a weaker liquidity effect on employment, output, consumption, and real deposits. As discussed by Heinemann and Marchlewitz (1996), a modified version of Christiano's model may exhibit an indeterminate steady-state and thus a continuum of stationary sunspot equilibria. In Heinemann and Marchlewitz model if the share of capital is around 0.45, capital depreciation is at least 23 per cent per quarter, and quarterly growth rate money supply seven percent, the model generates indeterminate solutions. Guo (1997) argues that the region of indeterminacy in Heinemann and Marchlewitz model is not plausible.[1] He shows that indeterminacy region depends crucially on the degree of returns to scale, inter-temporal elasticity of substitution and elasticity of labor-supply parameters. Moreover, he shows that this class of models low coefficient of risk aversion and high labor supply elasticity and sufficiently high increasing returns to scale can result in indeterminate steady state.

As mentioned before, two parameters are crucial for the workings of the model. One is the coefficient of relative risk aversion utility function (σ in equation 1)-the inverse of inter-temporal elasticity of substitution in consumption (IES). The higher is σ the more representative household is willing to defer consumption from one period to the next in response to an increase in the rate of interest. When there is a unit elasticity of substitution, the

1. See Guo (1997), page. 1.

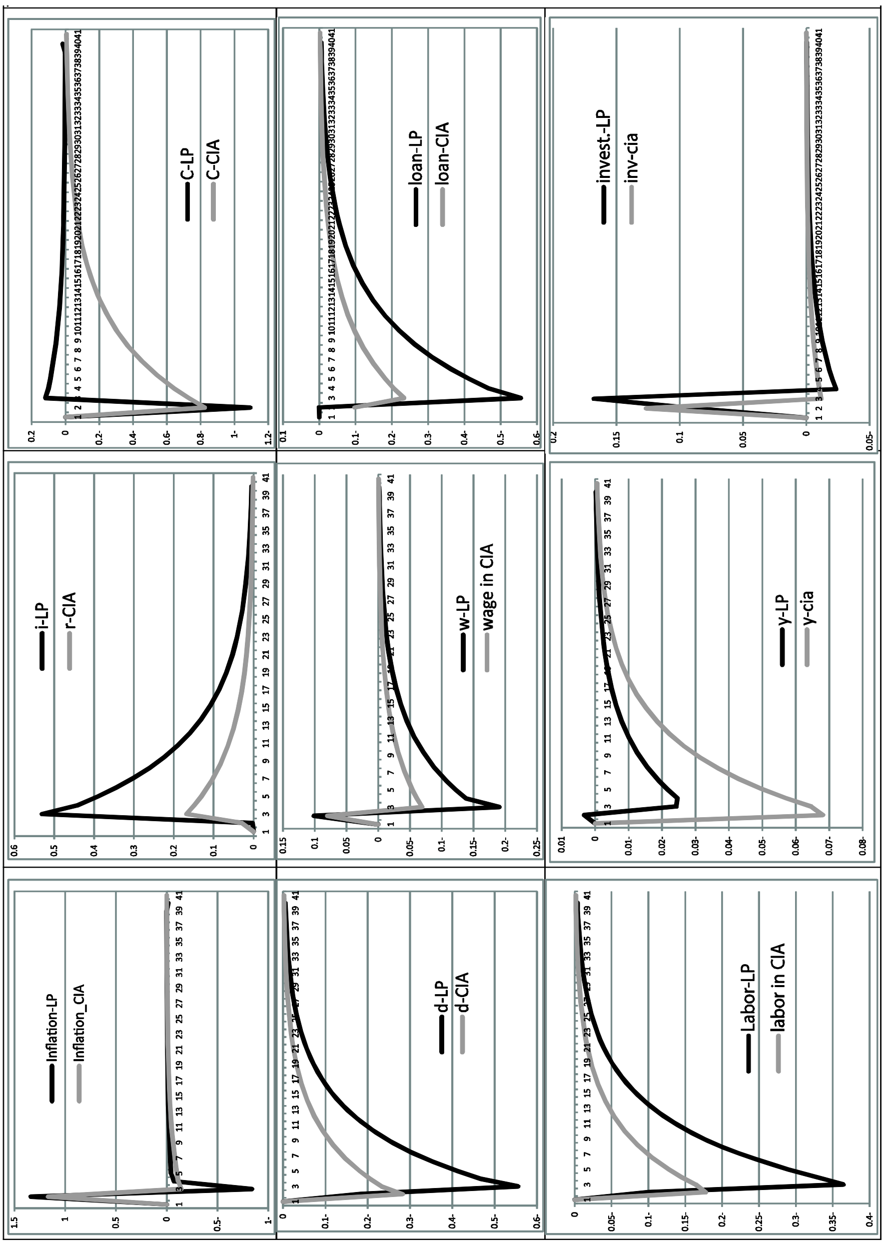
substitution effect is just strong enough to outweigh the income effect, so interest rate has no effect on savings. When IES is less than unity, such as in Iran, the substitution effect is not strong enough to outweigh the income effect and a rise in the interest rate reduces savings. There are different estimates of IES for Iran; in Jalali-Naini and Naderian (2011) the estimate is 0.46, in Shahmoradi et al., it is also estimated to be 0.46. This value shows that in the Economy of Iran the income effect of interest rate decline due to liquidity effect is greater than its substitution effect and it leads to a jump in real deposits.

The other important influence on the liquidity effect strength is in the elasticity of labor supply to real wages. When the parameter is lower than one the household's response to rise in real wage rate is smaller relative to the benchmark value of unity. This parameter is lower as well in the countries with low or middle income and rises as income increase. Frisch elasticity or the inverse of elasticity of labor supply to real wages (η) is estimated for Iran; in Taei (2006) the estimate is 2.1[1]. This value shows that the elasticity of labor supply to real wages is lower than unity and households do not increase their labor supply in response to real wage jump.

The sensitivity analysis performed on risk aversion coefficient in our model shows that when the coefficient of relative risk aversion is lower than one, we have an indeterminate solution. When we experiment with a Frisch elasticity value less than two the model also turns out to be indeterminate. A Frisch elasticity of higher than two results in a weaker liquidity effect on employment with no negligible impact on consumption.

1. Taei, Hasan (2006), Labor supply function: analysis based on Micro data, Iranian economic research quarterly, 29, 93-112

Figure (1): Impulse Response Functions for Lucas-Fuerst Model



Conclusion

In the basic cash-in-advance models (Cooley and Hansen), monetary versions of real business cycle models, expansionary monetary shocks are distributed proportionally to all economic agents and the resulting proportional increase in the price level does not alter agents' real money balances. In this class of models a positive monetary shock results in a rise in interest rates and a fall in output and employment because these models exhibit only the anticipated inflation effect (Christiano 1991). However, injection of reserves by the central bank can have a liquidity effect. Limited Participation models relax this assumption and by allowing for differences in the information flows between consumers, financial intermediaries, and firms, Lucas (1990), Fuerst (1992) and Christiano and Eichenbaum (1992a) introduce a liquidity effect which results in a non-neutral outcome.

In this paper we examined the impact of injection of liquidity to the banking system by the monetary authority, in a limited participation model, on inflation, output, consumption, employment, nominal interest rate, real money supply, real loans, loans, and real deposit. The impulse-response functions based on parameter values for the Economy of Iran do not show any liquidity effect. Simulation results indicate the dominance of inflationary expectations, resulting in a higher nominal interest rate, increase in real wage, fall in employment and output, and reduction in the volume of real money, real deposits, and real loans. The interpretation, given the structure and limitation of our model, is that a general injection of liquidity to the banking system (monetary base expansion) is not a reliable policy for raising output and employment. The inflationary expectation effect tends to dominate the interest rate effect and as a consequence output and employment suffer short-term contraction. The same result is obtained with different parameter values, except for the risk aversion parameter below unity and Frisch elasticity below two.

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