

# Inflation and Output in a Cash Constrained Economy<sup>1</sup>

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## Abstract

*We examine permanent effects of monetary expansion in an economy where access to credit for financing consumption and investment is limited and consumers and firms are cash-constrained. The main difference between our model with those of Cooley-Hanson (1989) and Walsh (2003) is that investment, in addition to consumption, is subject to a cash-constraint. In this respect, our model is similar to Stockman (1981) and Abel (1985) but different from them in that they do not provide for labor-leisure choice. Moreover, in contrast to Stockman and Abel we follow Svensson's (1985) timing sequence in that the asset market opens after the goods market. A version of Cooley and Hanson model is calibrated with the data on the economy of Iran. We compare the business cycles and output and consumption moments generated from simulated data to the moments extracted from the actual data. From the impulse-response functions we also derive the effect of a positive monetary shock on output and inflation.*

**Key words:** Inflation, output, growth.

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## 1. Introduction

The core issue of monetary theory is the articulation of the relationship among money, prices and real activity in the short and the long-run. A common objective of monetary policy is to maintain low inflation and promote high rates of economic growth (or keep economic growth close to the “potential” rate). A central assumption in the established monetary theory and a frequently cited stylized fact is that: in the long-run inflation is primarily a monetary phenomenon and the positive correlation between the two is stronger, the longer is the time horizon (King 2003). A negative relationship is also cited between inflation and GDP growth rate (Fisher 1993, Barro 1995, King 2003) but this relationship is less robust compared with the previous one and is subject to different interpretations. Mallik and Chowdhury (2001) found that in a number of South Asian countries the relationship is positive. To the extent that there is a negative relationship, it can be linear, non-linear or discernible above a threshold rate (Bruno and Easterly 1998, Khan and Senhadji 2001 and Sarel 1996). The empirical evidence in Iran tends to support the above generalizations in that there seems to be a significant positive correlation between monetary growth rate (M1) and the inflation rate (calculated from CPI) and that, at least in certain time periods, there is a negative correlation (albeit weak) between inflation and GDP growth rate and it is not robust to the sample selected (figure 1 and figure 2).<sup>1</sup>

Similar to the existing dissent regarding the empirical relationship between inflation and growth there are theoretical models that imply different relationships. Various models have been developed to determine the effect of monetary growth (via inflation) on economic growth and there is a vast literature on the long-run relation between inflation and capital accumulation. Within this collection, a number of papers stand out both in terms of their influence and implications regarding the effect of higher rates of monetary growth on economic

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1- Based on UK data King (2003) found that, in contrast to the close co-movement between money and prices, there is no a significant long-term relationship between growth rate of output and money.

growth and steady level of output. Assuming that individuals' savings out of current income consist of money or real capital, Tobin (1965) shows that a higher growth rate of "outside" money results in a higher inflation rate and a lower rate of return on money. This induces a portfolio shift out of money into capital. As a consequence, both capital intensity and steady state per capita output rise. Sidrauski (1967) shows that money is "super-neutral" in a utility maximizing framework of infinitely lived individuals—implying that steady state values of output and consumption per worker are not related to monetary growth rate (via inflation).<sup>1</sup> Fischer (1979) has demonstrated that for a class utility function in which real money balances and consumption are separable, inflation reduces consumption. Faria and Carneiro (2001) provide evidence in support of super-neutrality but also find support that high inflation affects short-run growth rate. A different set of conclusions is reached by Stockman (1981), Abel (1985) and Gomme (1999), in that an increase in the inflation rate results in a lower steady state level of output and consumption, if there is a cash-in-advance (CIA) constraint on both consumption and investment. Cooley, Hanson (1989) and Walsh (2003) also find that monetary growth has a negative effect on steady state output when CIA constraint only applies to consumption.

The theoretical conclusions regarding the direction of the permanent effect of monetary expansion on economic growth or steady-state output (the issue of super-neutrality or non-super-neutrality) is influenced by the presumed functions of money—that is how money is introduced into the economy—and the manner in which those functions are incorporated into the theoretical models. In particular, the results are sensitive to whether money is incorporated as a substitute or a complementary to capital. There are several methods of integrating money into dynamic general equilibrium models and examine the issue of super-neutrality. One way is to specify money as an argument in the utility function (Sidrauski 1967). An alternative is the shopping time or transaction cost approach (McCallum and Goodfriend, 1987). Another approach is to

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1- In Sidrauski's model, individual's saving ratio declines as a consequence of a higher inflation rate. "Indeed, people match their decline in saving dollar for dollar with a decline in money balances. Capital is unchanged..." Haslag (1997), p. 13.

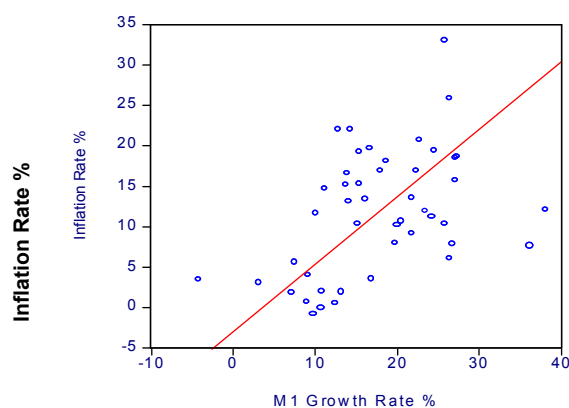
introduce money in a general equilibrium setting through the assertion of a CIA constraint (Clower 1967, Lucas 1982, 1987).<sup>1</sup> In this paper, we would like to focus on the latter approach because of the type of economic environment it describes. More specifically, we are interested in examining permanent effects of monetary expansion in an economy (like that of Iran) where access to credit for financing consumption and investment is limited and consumers and firms must have cash to carry out their consumption and investment plans. We would like to trace the effect of erosion of purchasing power of money resulting from the practice of inflationary policies on steady state output and consumption in the above-mentioned environment within the confines of a parameterized dynamic general equilibrium model with a CIA constraint.<sup>2</sup>

In section 1, we motivate a cash-constrained economy via transaction demand. Section 2 presents a cash-constrained infinite-horizon representative agent model with a period utility function specified in consumption and leisure with cash constraint on both consumption and investment. We will derive analytical solutions regarding the effect of higher inflation rates on steady state levels of consumption and output per worker. In section 3, we derive a utility maximizing money demand function from the model in section 2 and estimate the elasticity for intertemporal substitution in consumption (EIS). A crucial parameter that influences the dynamics of output in the aftermath of a monetary shock

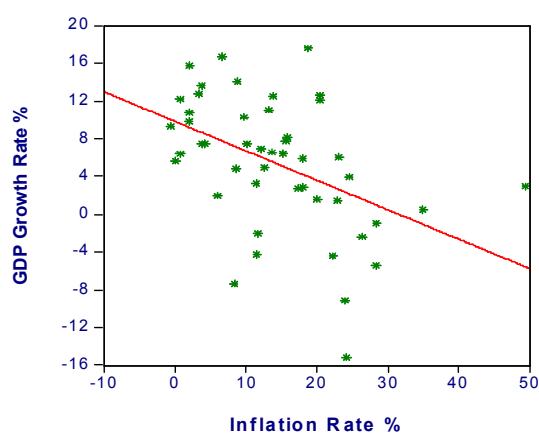
- 
- 1- Theoretical developments in this field since Hicks (1935) has had also to contend with, and provide an explanation to a classic question and two puzzles. Firstly why fiat money which is devoid of intrinsic value is exchanged with commodities that have use-value (yield utility), hence, have a positive price in terms of commodities. Secondly why money is the predominant medium of exchange (in numerous economies) when other assets (of similar risk class) are available to provide a positive rate of return? In the General equilibrium setting, a number of approaches have been developed to provide an explanation for these puzzles and a micro-explanation for holding (demand for) money. The utility approach and the cash-in-advance models are the widely used approaches to implant money onto general equilibrium models and to explain these puzzles. While the overlapping generation model can provide an explanation why individuals hold money, it requires a cash-in-advance specification to handle the dominance of the return puzzle rate. For more details see Farmer (1992).
  - 2- Our model is different from Stockman (1981) and Abel (1985) in that we allow for labor-leisure choice and also it is different from Gomme (1997) who establishes the relationship between anticipated inflation and output with general (unspecified) functional forms.

is the size of the EIS. In section 4, a standard cash-constrained, dynamic, stochastic and general equilibrium (DSGE) model calibrated with data from the Economy of Iran is simulated. We will compare the business cycles and output and consumption moments generated from simulated data with the moments extracted from the actual data. Moreover, we will examine the effect of a monetary shock on output and consumption and discuss the sensitivity of the dynamics of adjustments to the underlying parameters of the model and their implications. Section 5 presents the conclusion and policy suggestions.

**Figure 1: Inflation and money growth rates in Iran, 1960-2008**



**Figure 2: Inflation and GDP growth rates in Iran 1960-2008**



### **1. Transaction Demand and the Cash in-Advance Constraint**

The initial question for the dynamic general equilibrium models that integrate money is: why individuals hold money and why money has a positive value in equilibrium; given that, in its capacity as the store of value, money is dominated by interest-paying assets. The cash-in-advance literature provides an answer to this question by drawing on the function of money as a medium of exchange. The CIA approach (Lucas 1982, Svensson 1985, Lucas and Stokey 1987) stipulates that spending must be carried out with money and in this way motivates the transaction demand for money. Given that the liquidity services of money are required to implement transactions, a trade-off between holding money versus interest-bearing assets results. This provides an incentive for economically rational individuals to hold positive fiat-money balances which are intrinsically valueless and whose return is dominated by other assets in economy. The demand for money thus can be seen as a consequence of this trade-off which also determines the form in which a person's wealth should be held.

Transaction demand is motivated by positing that in every period a representative consumer makes a choice between money balances held, consumption and saving (asset holdings). In the basic model, all consumption goods must be purchased with money hence the consumer must have sufficient cash balances in advance. No interest is paid on the cash which is held in a bank. However, assets held by the individuals in the bank earn a positive interest rate. The return on money depends on price fluctuations—the purchasing power of money. The individuals choose the desired amounts of their consumption, money balances, and assets after they observe the state of the economy—for instance, once they observe the current period's money supply and value of the productivity shock. Markets open sequentially and there are two ways of looking at this sequence. Once the state of the economy is observed, the asset market opens first. Subsequently the goods market opens and the individual makes purchases of consumer goods through successive withdrawals of money. This is the Lucas version. In Svensson's (1985)

model, the goods market opens initially then the asset market opens. In this case, the consumers must carry over cash from previous period to purchase goods in this period.

## 2. The Model

In this section, we present a CIA model and obtain analytical solutions regarding the effect of monetary expansion on consumption and output. The model presented in this section is a certainty version of Cooley and Hansen (1989) and Walsh (2003) in which capital and investment decisions are introduced and a labor-leisure choice is specified in the utility function. The main difference of the model presented here with the above class of models is that investment, in addition to consumption, is subject to a cash-constraint and leisure is the only credit good in the model. In this respect, the model of this paper is similar to those of Stockman (1981) and Abel (1985) but is dissimilar to them in that they do not deal with the issue of labor-leisure choice. Moreover, in contrast to the above papers, we follow Svensson's (1985) timing sequence in that the asset market opens after the goods market.

The representative agent has a utility function as specified in (1)

$$U = \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\phi}}{1-\phi} + \psi \frac{(1-n_t)^{1-\eta}}{1-\eta} \right], \phi, \psi, \eta > 0 \quad (1)$$

Where  $c_t$  stands for real consumption,  $n_t$  is a fraction of the individual's total time budget supplied to market-activity,  $1-n_t$  is the fraction of time allocated to leisure time and  $\phi$  and  $\eta$  are the inverse of the intertemporal elasticity of substitution of consumption and leisure, respectively, and  $\psi$  is a parameter of the utility function. There are two constraints facing the agent: the cash in advance constraint and the budget constraint. CIA models try to capture the meaning of the Clower constraint by stipulating (not explaining) that a subset of goods is cash-intensive and can only be purchased by money (cash goods) and the

individuals need to hold monetary balances.<sup>1</sup> The other subset consists of goods that can be purchased without cash (no prior money balances required), called credit goods. Note that this kind of classification and the form of CIA constraint are somewhat arbitrary. The real cash constraint by the representative agent is given by (2),

$$c_t \leq \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t} = \frac{m_{t-1}}{\Pi_t} + \tau_t \quad (2)$$

$M$  is money holding,  $P$  is the price level,  $T$  stands for the transfer from the government,  $\Pi$  is equal to  $\frac{P_t}{P_{t-1}}$  and  $\tau_t = \frac{T_t}{P_t}$  is the real money transfer (and is equal to  $m_t - m_{t-1}$ ). Following the Svensson's (1985) assumption that the goods market opens first, the individuals carry over money balances from  $t-1$  in order to buy consumer goods in time  $t$  (hence  $M_{t-1}$  in the cash constraint). Moreover, due to the absence of uncertainty, the individual knows  $P_t$  at the time she chooses  $M_t$ , which is  $M_{t-1}$ . The representative agent's real budget constraint iterated forward by one period is given by (3).

$$\omega_{t+1} = f(k_t) + (1 - \delta)k_t + \tau_{t+1} + \frac{m_t + I_t b_t}{\Pi_{t+1}} \quad (3)$$

$\omega_t$  stands for the individual's real resource constraint. The real value of resources in (3) is the sum of income  $f(k_{t-1}) = y_t$  received during time  $t$ , the real value of the capital stock netted out for depreciation  $[(1 - \delta)k_{t-1}]$ , nominal interest income on government debt— $b$  stands for the stock of real government debt,  $I_{t-1} = 1 + i_{t-1}$  and  $i$  is the rate of interest.

At period  $t$ , the choice variables for the agent are  $c_t$ , real money balance ( $m_t$ ), real bonds ( $b_t$ ) and capital ( $k_t$ ). The individual's state at time  $t$  can be characterized by real resources  $\omega_t$  and real cash holdings  $m_{t-1}$ . The individual's choice problem is stated by the Lagrangian (4):

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1- The transaction technology can be thought as the first trip for a bank to get cash for free but the next one is prohibitively expensive.



$$L = \frac{c_t^{1-\phi}}{1-\phi} + \psi \frac{(1-n_t)^{1-\eta}}{1-n_t} + \beta \omega_{t+1}(k_t, b_t, m_t) + \lambda_t [(r_{t+1} + 1 - \delta)k_{t+1} + w_t n_t + \frac{m_{t+1}}{\Pi_t} + \tau_t + (1+i_{t-1})\frac{b_{t-1}}{\Pi_t} - c_t - k_t - m_t - b_t] + \mu_t (\frac{m_{t+1}}{\Pi_t} + \tau_t - c_t - k_t + (1-\delta)k_{t-1}) \quad (4)$$

The first order conditions (FOCs) yield equations (5-9):

$$c_t^{-\phi} = \lambda_t + \mu_t \quad (5)$$

$$\lambda_t + \mu_t = \beta [\lambda_{t+1}(r_{t+1} + 1 - \delta) + \mu_{t+1}(1 - \delta)] \quad (6)$$

$$\psi (1 - n_t)^{-\eta} = w_t \lambda_t \quad (7)$$

$$\lambda_t = \beta_t \left( \frac{\lambda_{t+1} + \mu_{t+1}}{\Pi_{t+1}} \right) \quad (8)$$

$$\lambda_t = \beta \left( \frac{1+i_t}{\Pi_{t+1}} \right) \lambda_{t+1} = \beta (R_t \lambda_{t+1}) \quad (9)$$

The production function for the representative firm is a Cobb-Douglas type, output per capita ( $y$ ) is related to per worker capital ( $k$ ), the fraction of the individual total time budget allocated to market-activity ( $n$ ) and a constant  $A$ .

$$y_t = A k_{t-1}^\alpha n_t^{1-\alpha} \quad (10)$$

The firms maximize profit subject to the technical constraint (10). The FOC conditions are as follows:

$$r_t = \alpha A K_{t-1}^{\alpha-1} N_t^{1-\alpha} \quad (11)$$

$$w_t = (1 - \alpha) A K_{t-1}^\alpha N_t^{-\alpha} \quad (12)$$

Equations (11) and (12) yield the levels of capital and labor the firms want to employ in order to maximize profit. For equilibrium in the labor,

capital, and bond market, demand and supply for factors and bonds must be equal. The equilibrium conditions are:

$$N_t (\text{demand}) = n_t (\text{supply}), K_{t-1} (\text{demand}) = k_{t-1} (\text{supply}), b_t = 0.$$

The government budget constraint follows the rule that the amount of net nominal transfer is equal to  $M_t - M_{t-1} = T$ . Households' cash constraint is given by  $P_t(c_t + I_t) = M_{t-1} + T_t$ .

Putting the equilibrium values of capital and labor market into (6) and (7), we can rewrite them as:

$$\lambda_t + \mu_t = \beta[\lambda_{t+1}(\alpha A k_t^{\alpha-1} n_{t+1}^{1-\alpha} + 1 - \delta) + \mu_{t+1}(1 - \delta)] \quad (13)$$

$$\psi(1 - n_t)^{-\eta} = (1 - \alpha) A k_t^\alpha n_{t+1}^{-\alpha} \lambda_t \quad (14)$$

The nominal money supply grows at the constant rate  $\theta$ , hence  $M_t = M_{t-1}(1 + \theta)$ , and in the steady state the rate of inflation is set equal to  $\theta$  ( $\pi^* = \theta^*$ ). The steady state values for the system are given below. By combining the steady state values for (6) and (8), we get the intra-temporal consumption-leisure choice condition as follows:

$$\psi(1 - n_s)^{-\eta} = \beta(1 - \alpha) A k_s^\alpha n_s^{-\alpha} \frac{c_s^{-\phi}}{\Pi_s} \quad (15)$$

By combining (11), (13) and (14) we obtain:

$$\beta^2 A \alpha k_s^{\alpha-1} n_s^{1-\alpha} = \Pi_s [1 - \beta(1 - \delta)], \quad \Pi_s = 1 + \theta_s \quad (16)$$

The equilibrium condition in the goods market in the steady state is

$$A k_s^\alpha n_s^{1-\alpha} = c_s + \delta k_s \quad (17)$$

Total differentiation of 15 through 17 yields:

$$\begin{bmatrix} \beta\alpha(1-\alpha)k_s^{\alpha-1}n_s^{-\alpha}c_s^{-\phi} & \beta\alpha(1-\alpha)k_s^{\alpha}n_s^{-\alpha-1}c_s^{-\phi} + \psi\eta\Pi_s(1-n_s)^{-\eta-1} & \phi\beta(1-\alpha)k_s^{\alpha}n_s^{-\alpha}c_s^{-\phi} - 1 \\ \beta^2 A\alpha(\alpha-1)k_s^{\alpha-1}n_s^{-\alpha} & -\beta^2 A\alpha(1-\alpha)k_s^{\alpha-1}n_s^{-\alpha} & 0 \\ A\alpha(\alpha-1)k_s^{\alpha-1}n_s^{-\alpha} - \delta & A(1-\alpha)k_s^{\alpha}n_s^{-\alpha} & -1 \end{bmatrix}$$

$$\begin{bmatrix} dk_s \\ dn_s \\ dc_s \end{bmatrix} = \begin{bmatrix} \psi\eta(1-n_s)^{-\eta}(d\Pi_s) \\ [1-\beta(1-\delta)](d\Pi_s) \\ 0 \end{bmatrix}$$

It can be shown that  $\frac{\partial k_s}{\partial \Pi_s} < 0$ ,  $\frac{\partial c_s}{\partial \Pi_s} < 0$ ,  $\frac{\partial y_s}{\partial \Pi_s} < 0$  and the effect of money growth on steady state employment ( $\frac{\partial n_s}{\partial \Pi_s}$ ) is ambiguous.

Therefore, an increase in the growth rate of money ( $\theta=\Pi-1$ ), hence, a higher inflation rate influences the steady state values of output, capital and employment ( $n$ ) is non-neutral.<sup>1</sup>Note that when CIA is extended to investment, the firms must have sufficient cash balances in order to acquire capital and in this sense money and capital become complementary. Since a higher inflation rate reduces the purchasing power of the stock of money, individuals (and firms owned by them) reduce their purchases of both cash goods and capital. Moreover, as Greenwood and Huffman (1987) show that the return to labor falls when the inflation rate increases and in response, the labor supplied falls. Furthermore, given the type of production functions used in our model, the marginal product of capital is positively related to the quantity of labor. As a result, when the quantity of labor supplied (=employed, in equilibrium) declines in response to a rise in inflation, the return to capital declines, resulting in lower steady-state quantities of capital, consumption and output.

When the cash-constraint only applies to consumption, the total differential of the system is given by the following system:

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1- For a general functional form derivation of the effect of higher inflation on capital, consumption, and output, see Gomme (2006).

$$\begin{bmatrix} \beta\alpha(1-\alpha)k_s^{\alpha-1}n_s^{-\alpha}c_s^{-\phi} & \beta\alpha(1-\alpha)k_s^{\alpha}n_s^{-\alpha-1}c_s^{-\phi} + \psi\eta\Pi_s(1-n_s)^{-\eta-1} & \phi\beta(1-\alpha)k_s^{\alpha}n_s^{-\alpha}c_s^{-\phi} - 1 \\ \beta^2 A\alpha(\alpha-1)k_s^{\alpha-1}n_s^{-\alpha} & -\beta^2 A\alpha(1-\alpha)k_s^{\alpha-1}n_s^{-\alpha} & 0 \\ A\alpha(\alpha-1)k_s^{\alpha-1}n_s^{-\alpha} - \delta & A(1-\alpha)k_s^{\alpha}n_s^{-\alpha} & -1 \end{bmatrix}$$

$$\begin{bmatrix} dk_s \\ dn_s \\ dc_s \end{bmatrix} = \begin{bmatrix} \psi\eta(1-n_s)^{-\eta-1}(d\Pi_s) \\ 0 \\ 0 \end{bmatrix}$$

In the dynamic model with the CIA constraint only on consumption, none of the steady state output-capital, capital-labor, consumption-labor, and consumption-capital (which equals money-capital ratio) ratios are dependent on nominal money growth rate ( $\theta=\Pi-1$ ). An increase in the growth rate of money supply will increase inflation rate (hence a tax on consumption), as a result the amount of labor supply and consequently output and consumption decline. This is the channel through which monetary expansion has real effect and thus is non-neutral.

In summary: when both investment and consumption are subject to cash-constraint, a higher steady-state inflation rate, due to a higher growth rate of money supply, results in lower steady-state output and consumption. The same result is obtained when the cash-constraint only applies to consumption, given that the cross partial restrictions for the production ( $F_{kn}$ ) and the utility function ( $U_{c1-n}$ ) is non-negative. The decline in long-run output due to higher money growth rate (inflation) is the result of the negative effect on employment and the non-positive effect restriction implied on capital as mentioned above.

Our results are different from a base case Cash-in-advance model (Walsh 2003). In the base-case model, the steady state consumption and capital levels are independent of nominal money growth rate (the rate of inflation rate) and in this sense, money is super-neutral. Moreover, the basic CIA model yields no interesting money demand properties and velocity behavior. It is an established stylized fact that money velocity varies over time and over business cycles. The constant velocity result is

mainly due to the way the money's function is asserted. There are two ways around the problem of constant velocity. One way is a la Svensson (1985) by introducing uncertainty into the model and on this basis motivating holding of precautionary balances. The other is by differentiating between cash and credit goods (Lucas and Stokey 1987), therefore, allowing variability between money and consumption. The model in the next section allows for variable velocity.

### 3. Demand for Money in the CIA Model

For qualitative analysis and simulations of the CIA model, we need to estimate a number of parameters and use them for simulations. One of the key parameters of the model is the IES which can be estimated from the money demand equation obtained from the model in section 1. The demand function is derived with the assumption that the CIA constraint is always binding and demand for cash is due to both investment and consumption good purchases. From the specifications for the utility and production functions and the FOCs (5-9), we can derive a parameterized utility-maximizing demand function for money, as in (18).

$$m_t = I_t + \left( \frac{(1-\alpha)y_t}{\psi (1+i_{t-1})n_t(1-n_t)^{-\eta}} \right)^{\frac{1}{\phi}} \quad (18)$$

This equation can be rewritten by replacing the rate of interest with inflation as:

$$m_t = I_t + \left( \frac{(1-\alpha)y_t}{\psi R_t (\Pi_t) n_t (1-n_t)^{-\eta}} \right)^{\frac{1}{\phi}}, R_t = \frac{1+i_{t+1}}{1+\pi_{t+1}}, \Pi_t = \frac{P_t}{P_{t-1}} = 1+\pi_t \quad (19)$$

The solved demand function in (18) (or in 19) is a combination of cash demand for investment and a complex term involving output and interest rate (inflation). Since we do not have quarterly data on private investment equations, (18) and (19) cannot be estimated. One way to tackle this problem to get investment out of the cash constraint, that is, to work with a model in which only the consumption good is subject to cash-in-advance constraint. As shown by Jalali-Naini and Naderian

(2010), in this case, the demand for real money balances is obtained from the FOCs of a certainty version of Cooley and Hansen (1989) which yields an equation similar to (18) except private investment ( $I$ ) drops out.<sup>1</sup> The log-linear form of the demand for money when consumption is the only cash-good is:

$$\log m_t = \Gamma [\log A - \log(\psi) - \log(R_t)] - \log n_t (1 - n_t)^{-\eta} + \Gamma [\log y_t - \log(1 + \pi_t)], \quad \frac{1}{\phi} = \Gamma \quad (20)$$

The money demand obtained in (20) exhibits interest rate elasticity and variable velocity.<sup>2</sup> Moreover, demand for money is inversely related to the duration of work-day. The latter argument does not appear in the usual empirical specifications of money demand. In the formulation represented by (20), the income and inflation rate elasticities are equal, hence a unit increase in income and interest rate leaves money demand unchanged. The demand function for money (20) is rooted in the Euler condition (13).<sup>3</sup> The elasticities for GDP and  $\Pi$  in the theoretical formulation (20) are equal to the inter-temporal elasticity of substitution

1-The exact shape of the demand for money is  $m_t = \left( \frac{(1-\alpha)y_t}{\psi(1+i_{t-1})n_t(1-n_t)^{-\eta}} \right)^{\frac{1}{\phi}}$ .

2- As already noted, in the most basic CIA model, velocity is a constant (unity). However, the data indicates substantial variation in velocity. As mentioned before, there are Lucas and Stokey (1987) and Svensson (1985) solutions to the problem. The issue of variable velocity has been handled in a number of papers. Cooley and Hansen (1995) endogenize money velocity in a monetary business cycle framework with shocks to the real and money sectors. Hromcova (1998) generates variable velocity with a model that combines a stochastic growth model with CIA constraint. By incorporating precautionary demand and "idiosyncratic consumption shocks" with aggregate uncertainty, Telyukova and Visschers (2009) can generate velocity volatility in the order of the magnitude that is suggested by the data. The paper by Mendizabal (2006) extends the CIA model to create a general equilibrium version of inventory demand (Baumol 1952) by altering the transaction technology assumption in the basic CIA models.

3- The standard version of Euler equation assumes the absence of liquidity constraint, and as a result the ability of agents to smooth consumption over time. When liquidity constraint is significant, the GMM method may not produce good estimates for the "deep" behavioral parameters. Anecdotal data indicates pervasive liquidity constraint in the Economy of Iran. Hard data is limited and scanty to offer a reliable estimate of the severity of liquidity constraint. We do not opt for a straightforward estimate of the Euler equation because we have the unobservable magnitudes for  $\mu$  and  $\lambda$ .

for consumption. One way to estimate this elasticity is to use GMM estimation of the Euler equation. The result of this estimation is shown in table (1). Data on real private consumption expenditures, as indicated by the ratio of consumption in consecutive periods ( $\frac{C_t}{C_{t-1}}$ ), indicates a significant degree of instability compared with data from countries with developed financial and lending markets. However, differences in the degree of instability are not very large<sup>1</sup>.

**Table 1: GMM Estimate of Money Demand**

C(1)[(X1 <sup>(2)</sup> )(X2-1)				
coefficient	Estimate	Standard Error	t-Statistic	probability
C(1)	1.0148	0.0270	37.583	0.000
C(2)	2.17	0.7044	3.0895	0.0034
S.E of regression 0.1773			Sum squared residual 1.415	
Durbin-Watson stat 2.37			J-statistic 0.028368	
Instrument list: C X <sub>t-1</sub> X <sub>2,t-1</sub>				
X1= seasonally adjusted private consumption, X2=return on capital				

#### 4. Monetary Shocks and Output Behavior

In section (2), the discussions were primarily focused on the steady state properties of the model, particularly the effect of an increase in long run inflation rate on steady state output. We are also interested to know about the dynamic behavior of the economy as it adjusts in response to exogenous shocks, e.g. a monetary or a productivity shock. One way to handle this question is to use a theoretical model and simulate it with the estimates of the parameters of the model to generate artificial data. One

<sup>1</sup> -Using Dynare for estimation of parameter values based on Bayesian methods result in similar values as calibrated one.

key parameter which we need to estimate is EIS. This can be obtained with a GMM estimate of the money demand (19). However, since we do not have data on quarterly private investments, equation (19) cannot be estimated. An alternative is to estimate the elasticity of inter-temporal substitution in consumption from a money demand function obtained under the assumption that cash-constraint applies only to consumers. Accordingly, we estimate a demand function in the form specified as in equation (20) in section (1).<sup>1</sup> We generate artificial data from a Cooley and Hansen (1989) type model calibrated with the data from the Economy of Iran. For validation of the model, we compare the statistical properties of the artificial data obtained from the model with those which are observed in the actual data. Moreover, we can use the theoretical model to answer a number of "what if" questions by changing some of its parameters.

We continue to assume that the nominal money supply grows at the stochastic rate of  $\theta$ .<sup>2</sup> In contrast to section 1, the production function is subject to a stochastic productivity shock  $z$  (more specifically,  $e^{z_t}$ ). The process that the exogenous productivity shocks follow is described by (21) and  $\rho$  is specified to be 0.6 in our base-case simulations. If the money supply growth rate at the steady state is  $\theta^*$  and the deviation of money growth rate around the steady state is  $u$  ( $u_t = \theta - \theta^*$ ), which follows a process described by (22):

$$z_t = \rho z_{t-1} + \xi_t \quad (21)$$

$$u_t = \gamma u_{t-1} + \zeta_t \quad (22)$$

Where  $0 < \rho < 1$ ; and  $0 < \gamma < 1$  are parameters and  $\xi, \zeta$  are white noises with zero mean and constant variances. By log-linearizing the system 5-9 around the steady state values and simulating the model, we

1- This is obtained by stipulating that the individual maximizes the expectation of

$$\text{equation (20)} \quad U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\phi}}{1-\phi} + \psi \frac{(1-n_t)^{1-\eta}}{1-\eta} \right]$$

2- The evolution of the money stock is given by  $M_t = M_{t-1} (1+\theta)$ .



can examine the effect of nominal monetary shocks on the endogenous variables. Our estimate of the exponent of capital in the production function is 0.54;<sup>1</sup> the GMM estimate of the inter-temporal elasticity of substitution in consumption for Iran is 2.17 ( $\phi=0.46$ ) and quarterly m1 average growth rate during 1996-2006 was calculated at 5.59 percent. We assumed a discount factor ( $\beta= 0.985$ ), depreciation rate of capital stock ( $\delta=0.025$ ), and the average work-day equal to 8 hours. We estimated the autoregressive parameter  $\gamma$  in equation (22) from a first-order auto-regressive regression of the average quarterly money growth rate (m1) in Iran for the 1986-2007 period. The estimated value and the persistence parameter ( $\gamma$ ) is 0.66. The standard deviation of money growth shock obtained from the residual of the above regression is estimated to be 1.35 per cent.

The evaluation of the simulation generally includes two kinds of analyses: unconditional analysis that consists of correlations and second moments of the series and conditional analysis that incorporates impulse response functions.<sup>2</sup> The moments obtained from HP filtered simulation of the linearized steady-state equation system and the moments obtained from the HP filtered actual data for the Economy of Iran are shown in table (2). The standard deviation of the HP de-trended consumption and output series are close to each other in actual data. The same can be observed with regard to the simulated data. Note that the simulated data volatility is less than that for the actual data.<sup>3</sup> The correlation between consumption and output in the artificial data is not high but is very close to the correlation in the actual data.

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1- The value is based on an estimate of the aggregate production function for Iran in Jalali-Naini (2007).

2- In the unconditional analysis, both productivity and monetary shocks are simultaneously present. In the unconditional analysis, we focus on only one type of shock at a time. In this paper, our focus is on the monetary shocks.

3- Note that the model used in this section does not account for terms of trade or real exchange rate shocks which can be significant in the Economy of Iran.

**Table 2: Comparison of the Actual and Simulated Moments**

	Actual Data %	Simulated Data %
GDP standard deviation (SD)	3.34	1.13
Consumption SD	3.32	1.03
GDP-Consumption Contemporaneous correlation	0.2117	0.210

Next, we obtain theoretical impulse-response functions from the linearized FOCs around the steady state. Christiano, Eichenbaum and Evans (2005) argue that the DSGE models are well suited to replicate the implied dynamics in the data—that is, the impulse response functions. We administer a one standard deviation monetary shock and examine the dynamics of adjustment. Figure 3 shows the deviation of output and inflation from the steady state. In this model, the short-run dynamics of output and consumption are not independent of monetary growth rate (inflation). Output and consumption depart from their steady-state levels because an increase in the expected future growth of the money supply increases the present rate of inflation. As inflation increases faster than money growth initially (figure 4), the economy experiences a decline in real cash balances which, in turn, reduces consumption. As a result, real consumption expenditures decline. The tax on consumption induces substitution toward leisure. Consequently, labor supply declines which results in contraction of output. The simulation results are not sensitive to the magnitude of the current money supply growth rate. The general profile of the impulse-response of output and inflation is not very sensitive to the exponent of capital in the production function ( $\alpha$ ), the capital-stock depreciation rate ( $\delta$ ), and the discount rate ( $\beta$ ). Our simulations with  $\alpha=0.467$ ,  $\delta=0.016$ , and  $\beta=0.991$  did not significantly change the profile of the deviation of output from steady state and the

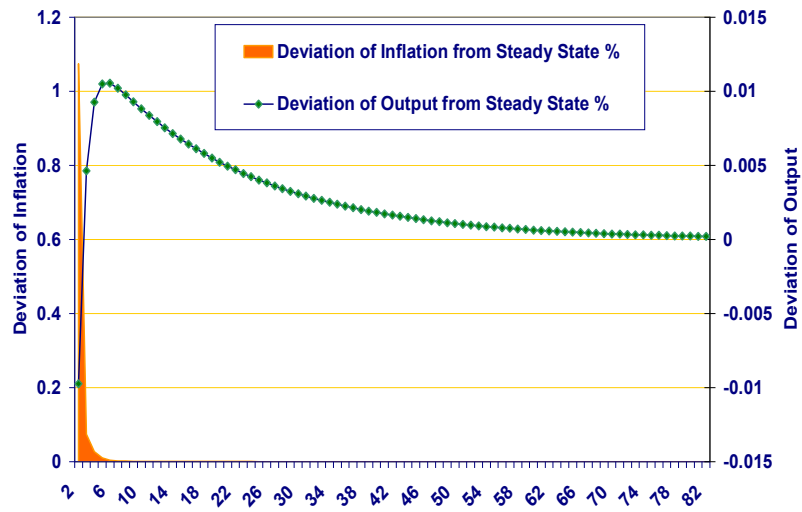
rate of inflation in response to a monetary shock.<sup>1</sup> In contrast, the magnitude of the elasticity of inter-temporal substitution in consumption influences the outcome of a monetary shock. The extent of deviation in output, consumption and employment and their dynamic adjustment in response to a shock increase in the money supply are sensitive to the magnitude of the EIS- as indicated in figure 5. When the inverse of EIS ( $\phi$ ) is low (say, 0.2), output initially falls below the steady state level then rises above it and finally reverts back to the steady state. When  $\phi$  is high (say, 2), the administered monetary shock results in an initial reduction in output but it subsequently rises above the steady state level. Low values for  $\phi$  result in both a transitional decrease and an increase in output relative to the steady state; in this sense the effect of a once-and-for-all positive shock in money supply on output is ambiguous. Note that this is due to the low value of  $\phi$ . Simulation with higher values shows that a positive once-and-for-all positive money shock results in the fall of output relative to the steady state level; as the effect of the shock dies out output reverts back to the steady state level.

Note that the adjustment processes exhibited through the theoretical impulse-response functions is very sensitive to the theoretical specification of the model. However, the impulse-response functions may not necessarily reflect the stylized facts in an economy. The theoretical model's findings should be validated by comparing it with the empirical impulse response functions generated by a VAR model. A number of empirical works in Iran indicate that an initial monetary shock has a positive output effect that dies out over time. The impulse-response functions depicted in figures 3 and 4 tend to be consistent with some of the VAR-based impulse response functions that examine the effect of a monetary shock on output and inflation in the Economy of Iran.<sup>2</sup>

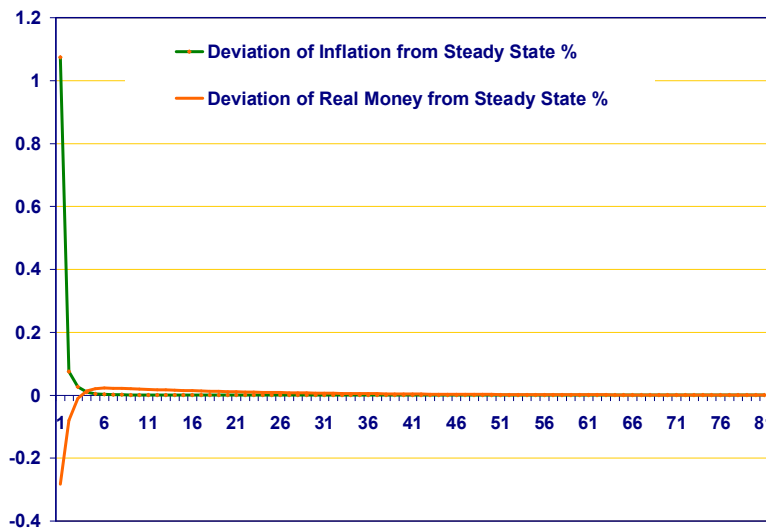
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- 1- The values for the above parameters were taken from Abbasinejad, H., A. Shahmoradi, and H. Kaavand (2009). The results of simulations with different parameters values are available upon request.
  - 2- See Shahmoradi et al. (1389), Jalali-Naini and Naderian (2010). Some VAR-based empirical impulse-response functions in the Economy of Iran, in contrast to Shahmoradi et al. (1389), do not show an initial fall in output; only a transient increase in output.

An additional factor that influences the adjustment process is the serial correlation of money growth process. The higher is the correlation of the money growth process, the larger is the impact of the expected future money growth on the present inflation rate. As a result, the initial decline in output is larger and lasts longer. The value of  $\gamma$  can be thought of representing the systematic component of monetary policy. If the economy is hit by a money growth rate shock with a high value of  $\gamma$ , say 0.9, the individuals will come to expect that higher money growth rates are going to persist for a while and as a result, inflationary expectations and hence the inflation rate will increase. The higher is  $\gamma$  the higher would be the increase in consumption tax and, as a result, the larger would be the substitution of labor for leisure and hence the decline in employment and output. In the case that  $\gamma=1$ , an initial shock to the money supply is repeated over time and the increase in the inflation rate becomes long-lasting. A permanent increase in inflation reduces output to a lower steady state level. In such a case, inflation remains higher and steady output remains lower (see figure 6). In contrast, when  $\gamma$  is zero, a positive money supply growth shock will be a one-time event—in contrast to an anticipated systematic increase. As a result, it has no effect on future inflation expectation: it only affects current inflation with no impact on output (figure 7) which points out an important aspect of the model: only anticipated monetary shocks have real effects. Unanticipated shocks only affect current inflation and have no real effects.

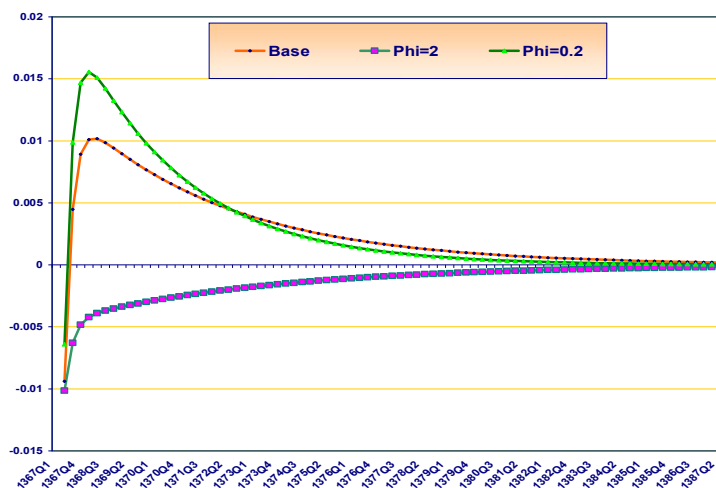
**Figure 3: Impulse-Response of the Monetary Shock**  
 (Base -Case values: average money growth rate=%5.59, coefficient of money growth persistence=0.66, capital share=0.55, discount factor=0.985, productivity shock=0, depreciation rate=%2.25 and elasticity of inter-temporal substitution in consumption=2.17)



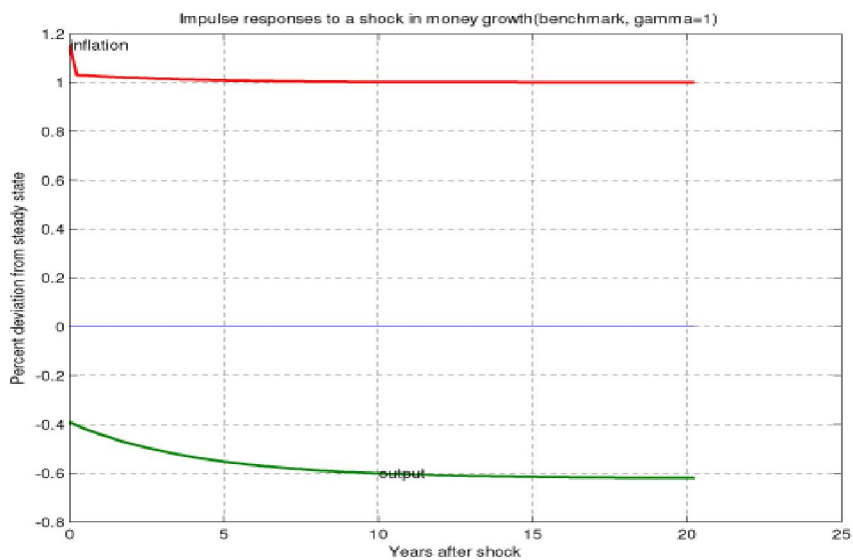
**Figure 4: Impulse-Response of the Effect of a Monetary Shock**



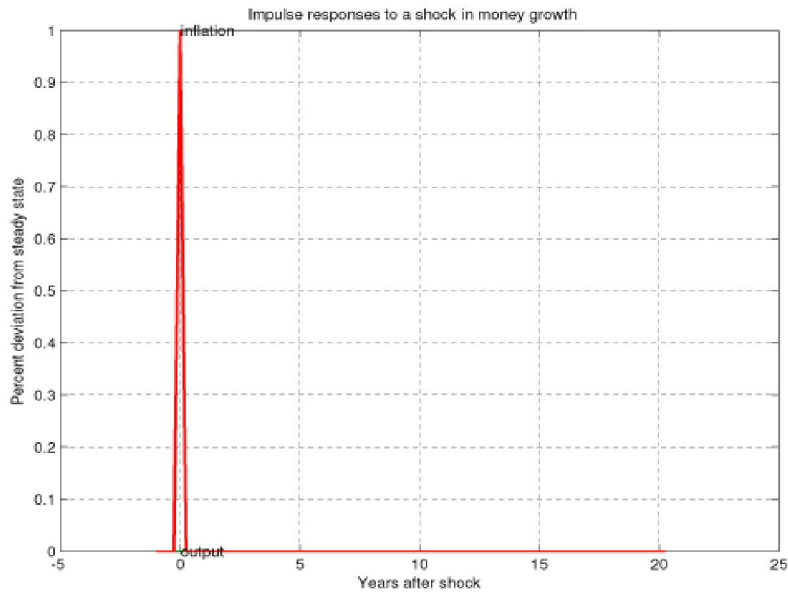
**Figure 5: Impulse-Response of the Effect of a Monetary Shock with different values for EIS**



**Figure 6: Impulse-Response of the Effect of a Monetary Shock with  $\gamma=1$**



**Figure 7: Impulse-Response of the Effect of a Monetary Shock with  $\gamma=0$**



## 5. Conclusion

There is no consensus in the theoretical works on the relationship between inflation and economic growth and the same can be said regarding the empirical findings on the subject. In this paper, we discussed this issue in connection with the question of super-neutrality of money and in the context of cash-in-advance models. We can conclude that when both investment and consumption are subject to cash-constraint, a higher steady state inflation rate, due to a higher growth rate of money supply, results in a lower steady state output and consumption. The same result is obtained if cash constraint only applies to consumption, given the non-negativity of cross-partial restrictions for both production and utility functions. The decline in long-run output due to a higher money growth rate (inflation) is the result of the negative effect of higher inflation rate on employment and the non-positive effect restriction implied on capital.

A Cooley and Hanson (1989) Walsh (2003) cash-in-advance model was calibrated with the data on the Economy of Iran. The statistical properties of the artificial data, generated through the simulation of the model, were compared to those for the real data. In particular, the contemporary correlation between consumption and output and their second moments was generated for the base-case parameters of the Economy of Iran. Moreover, the effect of a positive monetary growth rate shock on the endogenous variables of the model, in particular output, was derived from impulse-response functions. The dynamic adjustment of deviations of output from steady state in response to a once-for-all money growth shock heavily depends on the magnitude of the elasticity of inter-temporal substitution in consumption whose estimated value is relatively high in the Economy of Iran. With a high value of the EIS the cash-constrained economy experiences an initial decline but a subsequent rise in output relative to the steady state output, in response to a monetary shock. More specifically, for values of the inverse of EIS ( $\phi$ ) which are less than one, a positive money shock generates, both, a transitional decrease and subsequently an increase in output relative to the steady state; in this sense the effect of a once-and-for-all positive shock in money supply on output is ambiguous. Simulation with higher values shows that a positive once-and-for-all positive money shock results in the fall of output relative to the steady state level; as the effect of the shock dies out output reverts back to the steady state level. In addition, the parameter representing persistence in monetary growth significantly influences the dynamic adjustment to an exogenous monetary shock.

In this paper, it was shown that unanticipated monetary shocks had no effect on output; they only influenced the current inflation rate. However, a permanent increase in money growth rate (permanently higher inflation) results in a reduction in the steady state output. This implies that an anticipated shift from a low growth to a high money growth (inflation) policy has significant output effects because it upwardly revises inflationary expectations. In contrast, a one-shot increase in the money supply has no effect on output and only influences



the current inflation rate. The policy implication is that the monetary authority should adhere to a policy that anchors inflationary expectations.

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