Journal of Money and Economy
Vol. 20, No. 2, Spring 2025
pp. 287-346

Original Research Article

Unveiling the Hidden Symmetries in Financial Markets through Non-linear Analysis: Empirical Evidence from International Markets

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Received: 07 Jun 2025
Approved: 15 Sep 2025

This research examines the application of Chaos Theory and non-linear analysis in stock markets using empirical data from 20 international stock price indices spanning January 1984 to January 2024. A series of predictability and non-linearity tests provide strong evidence that stock price dynamics follow a non-linear data-generating process. Correlation dimension tests, implemented via the Grassberger–Procaccia algorithm, were employed to investigate low-dimensional deterministic dependence within the reconstructed state-space of the stock price index time series, yielding results consistent with chaotic behavior. Cumulative periodic tests were subsequently applied to further validate the presence of chaos in stock price dynamics. Following confirmation of predictability, ARFIMA, FIGARCH, LSTAR, and ESTAR models were utilized for out-of-sample forecasting over a clearly defined future horizon. Among long-memory models, the FIGARCH specification demonstrated superior forecasting performance by capturing both persistent volatility and long-range dependence. Among nonlinear models, the ESTAR framework exhibited the highest predictive accuracy. The findings offer valuable insights for investors and policymakers seeking a deeper understanding of the complex and chaotic nature of stock market behavior

Keywords: Chaos, Predictability, Non-Linearity, Market Indices
JEL Classification*:* C58, G10, G14, E44

[DOI: 10.66224/jme.20.2.275]

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1 Introduction

In financial markets, macroeconomic variables are often perceived as exhibiting irregular behavior and are accepted as random. Economists, however, linearize these variables using linear techniques that are relatively simpler in terms of inference. Chaos theory, which explains irregular behavior, offers a more realistic approach to modeling complex patterns of macroeconomic variables. Generally, chaos theory is a branch of mathematics that studies chaotic dynamic systems. Systems that become disordered may appear random but, in fact, follow patterns and deterministic rules that are highly sensitive to conditions. One of the applications of chaos theory is to explain the behavior of financial markets, such as the stock market. Indeed, the stock market and other similar markets are complex systems with chaotic behavior. In economics, money and financial markets are among the most suitable cases for applying chaos theory because if the process of determining monetary variables (Monetary variables in capital markets refer to quantitative measures that reflect the supply and demand of money and credit within these markets. They encompass various economic indicators that influence investment decisions and market behavior; such as VIX (Volatility Index), Liquidity Metrics: Indicators of market depth and trading efficiency, Credit Spreads: Differences in yields between corporate bonds and government securities, Interest Rates and Exchange Rates) follows a certain non-linear process, their changes can be predicted. If the final order in the process of monetary variables is discovered, it is possible to achieve profits. Regarding the non-linear relationship and chaos in stock indices, researchers have conducted several research studies (Vogl, 2024; Fang & Xu, 2003; Kazem et al., 2013; Shively, 2003; Alves et al., 2018; Abhyankar et al., 1995; Spelta et al., 2022; Wen et al., 2012; Shintani & Linton, 2004; Hsieh 1991; Klioutchnikov et al., 2017; Abhyankar et al., 1997; Small & Tse, 2003; Ozdemir & Cakan 2007; Salami, 2002; Moshiri & Forootan 2004).

Most researchers posit that financial markets operate according to non-linear processes (Thomaidis and Roumpis, 2010), which can be approximated through linear predictions, thereby offering a viable framework for forecasting the future trajectory of financial variables. Findings from Mramor & Kosta (1997) and Pahor & Mramor (2001) corroborate the existence of a non-linear relationship, challenging the conventional wisdom that stock returns are linearly related to financial variables. If predictability, in essence, is attained, the subsequent steps involve modeling the process, estimating model parameters, and making forecasts. Figure 1 illustrates the fluctuations of the

[DOI: 10.66224/jme.20.2.275]

global index. The confirmation of non-linear dynamics implies predictability in a structural sense, meaning that future price movements are not purely random but are partially governed by deterministic patterns embedded in the data-generating process. In this context, predictability does not suggest exact point forecasting; rather, it indicates the existence of exploitable temporal dependence and state-dependent behavior arising from non-linear interactions among market forces. Figure 1 depicts the long-term evolution of the Dow Jones Industrial Average (DJI), revealing a pronounced upward trend punctuated by abrupt regime shifts, volatility clustering, and sharp drawdowns most notably during periods of systemic stress such as the global financial crisis and the COVID-19 shock. These sudden collapses followed by rapid recoveries, along with alternating phases of calm and turbulence, are inconsistent with linear stochastic models and instead reflect hallmark features of chaotic systems, including sensitivity to initial conditions and non-proportional responses to shocks. The coexistence of long-run growth with intermittent, irregular fluctuations suggests that the DJI operates under a non-linear deterministic structure masked by apparent randomness. Consequently, Figure 1 provides visual and empirical motivation for adopting chaos theory and non-linear analytical frameworks to capture the hidden symmetries and dynamic instabilities governing international financial markets.

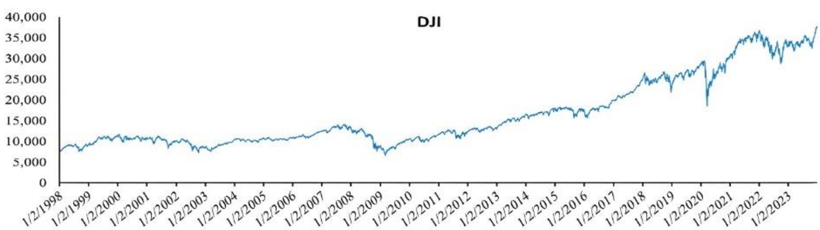

Figure 1. Trend Dynamics of the Dow Jones Industrial Average (DJI)
Source**:** Research findings

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Whaley (2009) provided further studies to generalize the concept of oscillation. The VIX index, developed in 1993 using stock options of the Chicago Board of Directors and modified in 2003 to reflect changing investor sentiment, is considered a promising economic indicator that reflects investor sentiment based on market participants' behavioral expectations. Whaley (2000) demonstrated that the VIX is a promising economic indicator, reflecting investor sentiments based on the behavioral expectations of market participants. Consequently, it is considered an essential indicator of future stock market risk. Therefore, it is crucial to pay attention to the VIX in research related to forecasting stock market trends, stock returns, and changes in VIX, as neglecting it can lead to errors in decision-making, management, and ultimately, policy recommendations. Previous research has examined the relationship between VIX indices and stock market fluctuations, as evidenced by studies conducted by Amengual and Xiu (2018), Magner et al. (2021), Huang et al. (2019), Whaley (2000), DeLisle et al. (2011), Liu and Fan (2013). In Figure 2 presents the empirical evolution of the Volatility Index (VIX), a paramount gauge of market risk and investor sentiment derived from S&P 500 index option prices. The plotted trajectory serves as a critical empirical backbone for our investigation into hidden symmetries and non-linear dynamics in financial markets. The observed time series is not a random walk

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but exhibits distinct regimes and punctuated, explosive spikes, most notably during the 2008 Global Financial Crisis and the 2020 COVID-19 pandemic shock, interspersed with prolonged periods of suppressed volatility. These structural breaks and transitions between high and low volatility states are quintessential manifestations of the non-linear, regime-dependent behavior theorized in our analysis. The apparent clustering of volatility and the asymmetry in its ascent and decay phases provide visual, empirical evidence of the underlying complex system's dynamics. This chart substantiates our core premise: that financial market volatility, as proxied by the VIX, embodies fractal patterns and hidden symmetries that are only decipherable through non-linear analytical frameworks, moving beyond traditional linear models to reveal the deep structural organization governing market stress and stability across international financial ecosystems..

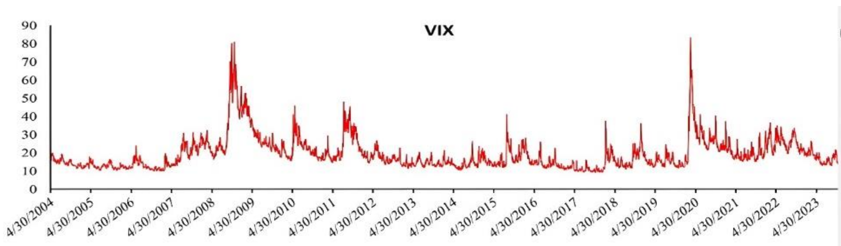

Figure 2. Time Dynamics of the VIX Index
Source: Research findings

The primary objective of this study is to examine the presence of chaotic dynamics in stock markets and their implications for understanding and anticipating market behavior, using a non-linear analytical framework that uncovers hidden structures and patterns within financial time series. Forecasting in this context aims to reduce investment and trading risk by identifying periods of heightened market volatility and irregularity, allowing market participants to adjust their strategies through tools such as volatility-based risk management, position sizing, and timing of trades. Although no prediction can be perfectly accurate, employing non-linear and chaos-based methodologies can improve the quality of forecasts by capturing complex interactions that traditional linear models may overlook, thereby enhancing decision-making under uncertainty. Numerous studies have been conducted in the field of forecasting stock market trends, including works by Li & Sun (2023), Liu & et al. (2022), Vogl (2024), Kazem et al. (2013) and Alves et al. (2018). The random walk theory posits that price changes from one transaction to another are independent of each other, and that trend analysis is an unreliable guide for determining future prices, making prediction impossible (Zakikhani, 2007). However, linear models based on time series econometric methodologies, in some cases, successfully provide short-term forecasts. Despite their reliance on a random process, long-term forecasts from such models are more prone to errors and deviations from the actual path.

Conversely, it is conceivable that the residuals in linear models are not random, a situation that contradicts the fundamental assumption of the randomness of these models' structure. This contradiction and deviation in analysis and prediction necessitate modifications to these models to enhance prediction accuracy. As a result, the research explores a novel methodology termed non-linear dynamic systems, specifically referring to chaos (Abrishami et al, 2002). In the current study, various tests are employed to investigate the predictability of market trends using the chaos approach. The primary challenge is identifying the means to detect hidden order in very chaotic complex systems. If such order is identified, it becomes feasible to predict fluctuations and market trends. From the perspective of chaos theory, complex systems may appear chaotic, irregular, and random, yet they are governed by a specific flow with a mathematical formula. This concept is often expressed as specified chaos, based on the theory of Nonlinear Growth with Feedback (NGWF).

The Problem Statement of this research revolves around the fact that financial markets are often perceived as exhibiting chaotic and random behaviors, which are generally modeled using linear techniques. However, these linear models fail to capture the complex, non-linear dynamics that govern market fluctuations. Despite evidence suggesting that financial markets follow non-linear processes, traditional models persist in dominating financial analysis. Chaos theory, which offers a more accurate representation of seemingly random systems by revealing hidden deterministic patterns, has not been fully applied to stock markets at the international level. Therefore, the challenge lies in detecting and understanding these hidden symmetries to improve predictive accuracy and risk management, particularly during volatile and crisis-stricken periods, which in this study are analytically defined as episodes of abrupt structural breaks, extreme volatility spikes, or major macroeconomic shocks captured through sudden jumps in stock indices and regime changes.

[DOI: 10.66224/jme.20.2.275]

The structure of this research is organized into several key sections: Section 1: Introduction, which outlines the research problem, objectives, and significance; Section 2: Literature Review, summarizing previous studies on chaos theory and its application in financial markets; Section 3: Methodology, detailing the research design, data collection, and analytical techniques used; Section 4: Empirical Analysis, presenting the findings of the study; and Section 5: Discussion and Conclusion, summarizing key findings, contributions, and suggesting future research directions.

2 Literature Review

The Literature Review of this study provides a comprehensive examination of the existing research on chaos theory and its application to financial markets. It explores the key theoretical foundations, models, and empirical findings related to the non-linear dynamics that govern market behavior. By reviewing a wide range of studies, this section highlights the gaps in the current literature, particularly the lack of thorough application of chaos theory to international stock markets, and establishes the theoretical framework that underpins the research. The review also delves into the key concepts relevant to the study, providing a solid foundation for the research's theoretical approach.

2.1 Chaos Theory and Non-linear Processes

Chaos theory examines how seemingly unpredictable systems can exhibit hidden structures and regularities. A chaotic process is the product of a nonlinear dynamic system; most researchers concur that financial markets operate on a non-linear process. A chaotic process is the product of a nonlinear dynamic system; most researchers concur that financial markets operate on a non-linear process (Thomaidis & Roumpis, 2010). Consequently, linear models may not yield suitable results for forecasting the future trajectory of financial variables. These systems have been observed in both natural phenomena and human behavior. A distinctive feature of chaotic dynamical systems is their deterministic nature, despite being sensitive to initial conditions (Altan et al., 2019). This sensitivity means that even minor fluctuations or changes in initial conditions can prevent precise long-term predictions. However, according to chaos theory, if a system is examined over a period of time, observing the system's states at different moments reveals

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the inherent order within the system. Even the most unpredictable (chaotic) systems operate within certain boundaries, never deviating from them. Within apparent disorder and chaos, there often exists a pattern of order that is surprisingly beautiful (Tehrani et al., 2011). In fact, a system with a chaotic process generates oscillations with an essentially infinite period. Conversely, a chaotic system creates cycles that never repeat within the studied period. Non-repetitive cycles in a chaotic system arise due to its nonlinear boundaries, leading to movements that stretch backward and forward, not aligning with previous paths. These forward and backward movements make a chaotic system sensitive to its initial conditions. If one or more values of the initial conditions change very slightly, the new system's time path will exponentially diverge from its previous path. This sensitivity is a critical feature of chaotic systems, prompting the use of tests to identify chaotic processes. The following section presents five key characteristics of chaotic processes.

2.2 Key Concepts in Chaos Theory

In this section of the paper, we explore the core concepts of chaos theory, including strange attractors, self-similarity, sudden structural breaks, the butterfly effect, and dynamic adaptation. These concepts form the foundation for understanding the behavior of chaotic systems and their application in complex systems, such as financial markets. By delving into these principles, we aim to provide a clearer understanding of the intricate dynamics that govern both natural and financial phenomena.

2.2.1 Strange Attractors

In the realm of chaotic systems, attractors exhibit a complex and enigmatic nature. A complex attractor is defined as an uncountable set of points within which all time paths that began inside it remain, while all time paths adjacent to it are drawn towards it. Time paths initiated within such an attractor can either be non-intermittent or can repeat in a predetermined number of instances. Unlike other attractors, which possess some degree of order and predictability, strange attractors are characterized by their disorder, hence the term "strange attractors." These attractors are perceived as chaotic from certain perspectives but exhibit order from others. If the scope of observation is expanded, it becomes possible to identify strange attractors, thereby enhancing predictive capabilities.

2.2.2 Self Similarity

In the realm of chaos theory, a remarkable form of similarity exists where each part of a system mirrors the entirety, revealing patterns that are identical across

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scales. This concept, known as self-similarity, is akin to shattering a mirror; each fragment is a reflection of the entire mirror, highlighting the system's ability to replicate its structure at various levels (Alvani & Danayi Fard, 1999). This principle underscores the complexity and order inherent in chaotic systems, offering insights into the intricate patterns found in nature and financial markets.

2.2.3 Sudden Structural Breaks in the Time Path

Chaotic series may exhibit sudden structural breaks at various stages of their time series. This phenomenon suggests that the behavior of a chaotic series is fundamentally distinct from that of a random series. A chaotic series indeed follows a specific process, characterized by large random disturbances occurring at random intervals. If the behavior of a series originates from a random process, it is not predictable. However, if it arises from a chaotic process, despite its complexity and apparent randomness, it is predictable due to the deterministic nature of the process (Moshiri, 2002). Figure 3 vividly illustrates the phenomenon of sudden structural breaks within the DJI time path, characterized by abrupt, high-magnitude shifts in volatility that punctuate periods of relative stability. These discontinuous jumps and regime changes are quintessential manifestations of chaotic dynamics in financial markets, reflecting the system's inherent non-linearity and sensitivity to external shocks or internal critical thresholds. The irregular and unpredictable timing of these breaks, alongside their fractal-like occurrence across different time scales, aligns with the core tenets of chaos theory, such as the Butterfly Effect and strange attractors, where deterministic, non-linear systems produce seemingly random, aperiodic outcomes. This pattern underscores that market evolution is not merely stochastic but is driven by complex, adaptive dynamics, where structural breaks represent pivotal transitions between different chaotic attractors, thereby revealing the hidden, discontinuous order within the apparent market turbulence.

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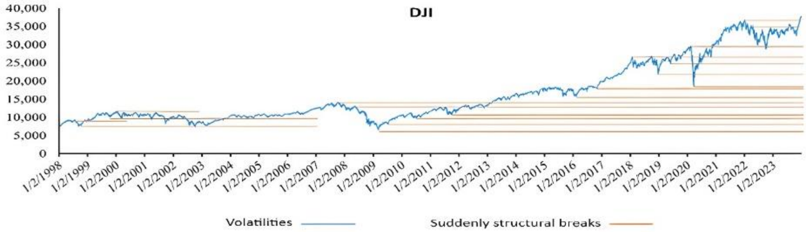

Figure 3. Structural Breaks in the Time Path of the Dow Jones Index
Source: Research findings

2.2.4 Butterfly Effect

Many natural phenomena exhibiting chaotic time series are highly sensitive to initial conditions. This means that two time series with chaotic processes but very close initial conditions will diverge significantly over time, presenting as entirely distinct time series. The closer the initial conditions of two series are, the longer their time paths remain similar (Baumol & Benhabib, 1989). Numerous studies in the financial field demonstrate that even minor or transient changes in the initial conditions of a system can lead to substantial, unpredictable shifts in future outcomes. This phenomenon underscores the inherent chaos of the world in which we live, making accurate prediction nearly impossible. The stock market exemplifies this chaotic nature. The Lyapunov exponent test can be employed to measure the butterfly effect, illustrating the sensitivity of chaotic systems to initial conditions.

2.2.5 Dynamic Adaptation

Disordered systems exhibit behaviors akin to those of living organisms in relation to their environment, creating a dynamic compatibility between themselves and their surroundings. The extent and manner of this adaptation are not predetermined and no pre-planned strategy for it exists; rather, it is an emergent and unplanned phenomenon that evolves over time. In the conceptual model presented in Figure 4 in the subsequent section, we aim to clearly and succinctly describe all aspects of a chaotic system. This Figure synthesizes the core concepts and methodologies of chaos theory and non-linear dynamics as applied to financial market analysis. It outlines a suite of diagnostic tools—such as the Lyapunov exponent, BDS test, Hurst exponent, and models like LSTAR and FIGARCH—used to detect hidden order and deterministic chaos within seemingly erratic market data. The figure underscores the fundamental principle that chaotic processes, characterized by sensitivity to initial conditions (the Butterfly Effect) and strange attractors,

[DOI: 10.66224/jme.20.2.275]

originate from non-linear dynamical systems. By employing these non-linear analytical techniques, the research aims to unveil latent symmetries and predictable structures in international financial markets, thereby providing empirical evidence that market fluctuations, though complex, may contain decipherable patterns for improved trend prediction and risk mitigation.

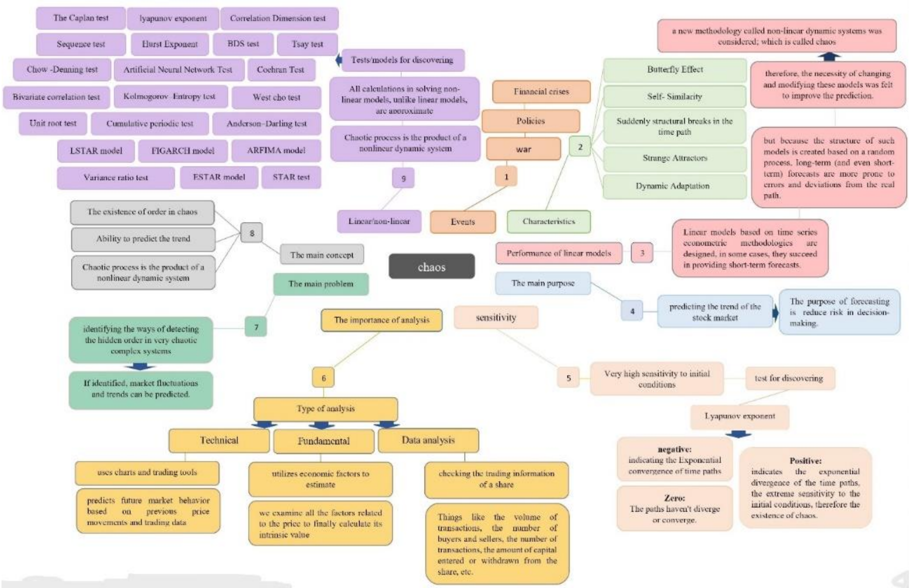

Figure 4. A Conceptual Chaos Model for Financial Markets
Source: Research findings

2.3 Contributions and Advancements to Financial Market Forecasting through Chaos Theory

This study makes a significant contribution to the existing literature by addressing a critical gap in the application of chaos theory to international stock markets. While previous research has investigated non-linear dynamics and chaotic behavior in financial markets, most studies still rely on linear

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models, overlooking the complexity and chaotic nature of market fluctuations at a global level. By applying a non-linear dynamic systems approach, specifically chaos theory, this paper introduces a new methodology for modeling and predicting stock market trends across diverse international markets. The research not only fills a crucial gap in financial market analysis but also offers significant advancements in forecasting market behavior, particularly during both stable and volatile conditions. Additionally, it provides new insights into the potential of chaos theory to enhance the accuracy of financial risk management practices, thereby making an impactful contribution to both theoretical frameworks and practical applications in the field of financial forecasting.

2.4 Efficient Market Hypothesis Against Chaotic Processes

Markets exhibit varying degrees of efficiency, with some information leading to efficiency and other information to inefficiency. Fama (1970) categorized stock markets into three levels of efficiency: poor efficiency, where past trends in stock prices are reflected in prices, and no individual with this information can earn a higher return than others; semi-strong efficiency, where public information, including past stock price trends, is reflected in prices, and no one can claim superiority in investment selection and higher returns; and strong efficiency, where stock prices reflect all information, including confidential information, preventing any individual from having an advantage in investment selection. This research primarily focuses on information efficiency, which is the third form of market efficiency in Fama's (1970) classification, commonly referred to as semi-strong efficiency. Information efficiency occurs when all publicly available information is fully and promptly reflected in asset prices, enabling the market to quickly adjust to new information. In this context, the study examines how the rapid dissemination and incorporation of relevant market information contribute to the efficiency of stock prices. Contrary to the efficient market hypothesis (EMH), which assumes complex time series, such as market prices, are random and thus unpredictable, research has rejected market efficiency at the weakened level (Hkiri et al., 2021; Elango & Hussein, 2008; Robinson, 2006; Ahmadzadeh et al., 2014). In this context, we discuss two characteristics of chaotic systems that contradict EMH.

2.5 Predictability of market returns against EMH

The theoretical basis for predicting the prices of various assets and market trends traditionally hinges on not accepting the efficient market hypothesis

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regarding market pricing (Özer & Ertokatlı, 2010). Conversely, based on the theory of chaos, a new hypothesis, known as the fractal markets hypothesis, is proposed, which opposes the efficient markets hypothesis to explain phenomena in financial markets. The validation of the fractal markets hypothesis relies on performing the largest lyapunov exponent test to ensure the predictability of the studied series based on non-linear models, and investigating the reverse largest Lyapunov exponent to determine predictable time horizons.

2.6 The Existence of Long-Term Memory Against EMH

The existence of long-term memory in financial markets is a pivotal theoretical and empirical issue. If markets exhibit long-term memory, there will be significant autocorrelation between observations spanning a very long period. Since these series are not independent over time, understanding the distant past aids in predicting the future. The presence of long-term memory in financial markets, as evidenced in the current research focusing on the stock market, contradicts the weak form of the market efficiency hypothesis and challenges the linear models of asset pricing. This suggests that non-linear models should be employed in forecasting and pricing capital assets.

2-7. Research Background

Table 1 presents an overview of the background and key concepts of non-linearity and chaos in financial markets, summarizing the theoretical foundations, empirical evidence, and methodological approaches used to analyze complex and chaotic market behaviors. This table provides a concise reference for understanding how non-linear dynamics and chaotic patterns have been studied and applied in financial market research.

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Table 1. Background of Non-Linearity and Chaos in Financial Markets

Author(S)	**Title Of Article**	**Year**	**Result**
Vogl	Chaos Measure Dynamics in a Multifactor Model for Financial Market Predictions	2024	time-variation of the chaos measure series, indicating chaos instability or inherent chaotic time variations of the underlying (hyper-) chaotic deterministic S&P500 return system.
Spelta et al	Chaos based portfolio selection: A nonlinear dynamics approach	2022	Results show that the CCP overwhelms several competing alternatives, both in terms of net profits and risk-return profiles.
Ataei et al	Theory and applications of financial chaos index	2021	computational results which pertain to the time period from January 1990 to December 2019 imply that there exists a two-way causal relation between the processes underlying the
Enayati Taebi	Chaotic Test and Non-Linearity of Abnormal Stock Returns: Selecting an Optimal Chaos Model in Explaining	2021	Results of these tests represented a non-linear and non-random process and chaos in the abnormal stock returns, implying the predictability of abnormal stock returns. Also, among
Bulusu et al	Near Future Stock Market Forecasting Based On Chaos Theory, Sentiment Analysis, and	2020	Back tests of fractal analysis and predictive algorithm produced significant results with considerable accuracy.
Aghasi and jozdani	The application of chaos theory analysis in the analysis of financial forecasts	2020	Price change in financial markets is one of the important variables that can have a significant impact on the economy of countries, managing and estimating the financial risk of stock
Alves et al	Detecting chaos and predicting in Dow Jones Index	2018	The study of time series formed by distinct periods reveals the chaotic dynamic of Dow Jones Index evolution.
Sahni	Analysis of Stock Market Behavior by Applying Chaos Theory	2018	As it can be inferred, significant trends can be identified from a stock's past performance.
Beygi and Abdolvand	Stock Price Prediction Modeling Using Artificial Neural Network Approach and Imperialist Competitive Algorithm	2017	According to the existing non-linear relationships between the variables affecting the stock price, artificial neural networks are one of the most suitable approaches for stock price
hashemi and horri	The survey of the chaos equations on Stock Exchange system (Case study: Tehran Stock Exchange system)	2016	The obtained results indicate that the Tehran Stock Exchange system follows a chaotic system.
Namazi et al	The study of the chaos process phenomenon in the price index and cash yield in Tehran Stock Exchange	2015	The results of the research indicate that the price index and cash yield are a chaotic and deterministic process.
Imamverdi and Safarzadeh	Chaotic test and non-linearity of stock price index in Tehran Stock Exchange	2015	The stock price index follows a non-linear process. Among the non-linear models, the ESTAR model has a higher predictive power

Source: Research findings

3 Methodology

This section presents the methodological framework of the study, which incorporates sophisticated measurement methods, including the Lyapunov exponent, Hurst exponent, and the BDS test. These tools were instrumental in demonstrating the models and criteria of the chaotic process, facilitating the validation of the non-linear trend and the identification of the most stable model for long-term predictions.

The research employs Chaos Theory and non-linear analysis within the stock market, leveraging empirical data from international markets to scrutinize the presence of chaotic trends and non-linear processes within the time series of 20 international stock price indices, spanning from January 1984 to January 2024. Through the application of predictability and non-linearity tests, the study uncovers evidence of a non-linear process in the stock price index. Additional tests, such as correlation dimension tests and cumulative periodic tests, are conducted to evaluate the correlation between observations and refine the analysis, acknowledging the chaotic nature of the stock price index variable.

Following the validation of the stock price index's predictability, a set of models was employed to analyze and forecast future market behavior. Linear and long-memory properties were examined using ARFIMA, which allows for modeling persistence in time series data. Conditional heteroskedasticity and time-varying volatility were captured using the FIGARCH model, which accommodates both long-term dependence and dynamic variance patterns. To account for non-linear and regime-dependent dynamics, threshold autoregressive models, including LSTAR and ESTAR, were implemented, enabling the identification of state-dependent responses in market movements. Each model was calibrated according to its respective estimation procedures, and all were applied systematically to the reconstructed stock price series, providing a robust methodological framework for capturing the complex linear and non-linear features inherent in financial market fluctuations.

3.1 Definition of Chaotic Variables

Table 2 presents an overview of the key variables used in analyzing chaotic behavior in financial markets. These variables include the Total Index (TI), Volatility Index (VIX), Stock Return (SR), Price of Stock Index (PI), and Volume of Stock Index (Vol). The table provides the definition and time period for each variable, which varies for each country, with the trading days ranging from a minimum of 5040 to a maximum of 9360. The data for these variables are sourced from public websites, such as Yahoo Finance and

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Google Finance, while stock return data is derived from the authors' findings. This information is essential for understanding the dynamic and potentially chaotic nature of financial markets over the given time periods.

Table 2
Definition of Chaotic Variables

Variable	Definition	Full period	Trading days	Data source
TI	Total index	Different for each country At least 1984/2/4–2023/12/28	Minimum: 5040 Maximum: 9360	Public website**
VIX	The volatility index	2004/4/3–2023/12/28	4333	Public website**
SR	Stock return	Different for each country	Minimum: 5040 Maximum: 9360	Authors' findings
PI	Price of stock index	Different for each country	Minimum: 5040 Maximum: 9360	Public website*
Vol	Volume of stock index	Different for each country	Minimum: 5040 Maximum: 9360	Public website*

Source: Research findings
Note: This table reports the information of chaotic indexes. The list of websites from which sources are extracted: *website 1 (https://finance.yahoo.com),**website 2 (https://www.google.com/finance)/(google sheets).

3.2 Rationale Behind the Application of Non-Linear Methods: ESTAR, ARFIMA, FIGARCH, and LSTAR

In this research, we employ four advanced non-linear modeling techniques: ESTAR (Exponential Smooth Transition Autoregressive), ARFIMA (Autoregressive Fractionally Integrated Moving Average), FIGARCH (Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity), and LSTAR (Logistic Smooth Transition Autoregressive) to capture the intricate non-linear dynamics inherent in financial market data. Each of these methods is chosen for their ability to model complex behaviors that linear models are often unable to represent.

1) ESTAR (Exponential Smooth Transition Autoregressive Model): This method is particularly well-suited for modeling data that exhibit smooth transitions between regimes. In financial markets, transitions between different market conditions (such as bullish and bearish periods) are often

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gradual rather than abrupt, and ESTAR effectively captures these dynamics by modeling the smooth change in the market's behavior over time. The non-linear structure of ESTAR allows us to observe how the market responds to external shocks and volatility in a continuous, yet non-linear manner, making it an ideal choice for time-series data that exhibit such transitions.

2) ARFIMA (Autoregressive Fractionally Integrated Moving Average): ARFIMA is used in this study due to its ability to model long-memory processes that are common in financial time-series data. Unlike traditional ARMA models, ARFIMA allows for fractional differencing, which means it can capture both short-term and long-term dependencies in financial data. This is particularly useful in financial markets, where past shocks can have persistent effects on future market behavior, even over long periods. ARFIMA's flexibility in handling fractional integration makes it a powerful tool for understanding and forecasting the behavior of financial indices that exhibit such long-memory characteristics.
3) FIGARCH (Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity): FIGARCH is employed to model the volatility clustering often observed in financial markets. This technique is designed to capture the non-linear relationships between past volatility and future volatility, a common phenomenon in financial markets where periods of high volatility tend to be followed by more periods of high volatility. The advantage of FIGARCH over traditional GARCH models is its ability to model long-range dependence in volatility, making it particularly effective for forecasting market risk and measuring the persistence of volatility shocks over time. This method is crucial for assessing the stability and risk associated with financial markets under various conditions.
4) LSTAR (Logistic Smooth Transition Autoregressive Model): LSTAR is a non-linear model that is particularly useful for detecting non-linearities in time series data where there are abrupt changes or shifts in the underlying structure of the data. In financial markets, these shifts often manifest as sudden market crashes, rebounds, or transitions between stable and unstable regimes. LSTAR is capable of capturing these regime shifts through its logistic function, which allows it to model the market's response to various internal and external factors in a way that traditional linear models cannot. By incorporating LSTAR, we are able to analyze and predict market behavior in the presence of sudden shifts or volatility.

[DOI: 10.66224/jme.20.2.275]

3.3 Nonlinear Tests and Predictability of Time Series

3.3.1 The Kaplan Test

Although this test is used to check the existence of stochastic linear processes against non-linearity; however, it has been discussed in chaos literature. Kaplan uses the fact that determinable paths, unlike stochastic processes, use the property that "points that are close to each other in the fuzzy space are also close to each other". Kaplan's statistic is denoted by $\mathrm{K} = \mathrm{E(r)}$. In fact, the Kaplan test says:

$$lim\ k = \lim_{r\to\infty} E(r) \begin{cases} > Kaplan\ table\ statistic^{*} \Rightarrow Linearity\ is\ not\ rejected \\ < Kaplan\ table\ statistic \Rightarrow Linearity\ is\ rejected \end{cases} \quad (1)$$

*Kaplan based his table on N (usually N=20) linear replaces which have the same histograms and autocorrelation functions as the observed data. The statistic in the table shows the minimum value that K must have (in linear mode). to calculate it, we have to choose the average K obtained from N replaces. for further reading, refer to Kaplan, 1994.

3.3.2 Runs Test

One method for testing the presence of chaos in time series involves assessing the serial dependence of returns. Serial correlation pertains to the correlation of successive returns over time. Among the tools used to identify serial dependence is the runs test, which is a non-parametric test. This test involves listing the number of consecutive positive and negative returns of the investigated variable and, conversely, testing and comparing the randomness hypothesis of the sample distribution. In this test, if the series of positive and negative returns exceeds the expected series, it indicates the existence of a non-random pattern in the stock price trend. Specifically, when the sign of autocorrelation changes between the residuals of the time series, a new series is formed. The total number of positive and negative cases created in the series is counted, followed by calculating the expected number of series and their standard deviation through a specific formula.

$$E(R) = 1 + \frac{2(n_1)(n_2)}{n_1+n_2} + 1$$ The number of dynasties (2)

$$\delta = \sqrt{\frac{2n_1\, n_2[2(n_1\, n_2) - n_1 - n_2]}{(n_1+n_2)^2\,(n_1+n_2-1)}}$$ Standard deviation of the dynasty (3)

or

[DOI: 10.66224/jme.20.2.275]

$$runs \sim N\,(\mu, \sigma^2)$$
$$\mu = \frac{2N^+N^-}{2!} + 1 \qquad (4)$$
$$\sigma^2 = \frac{(\mu-1)(\mu-2)}{N-1}$$

n= total sample size, n1= the number of observation of one type, n2= the number of observation of other type.

To determine whether the order of data is random, compare the p-value to the significance level. Usually, a significance level of 0.05 works well. p-value > 0.05 = H0 Rejected and H1 Accepted, p-value < 0.05 = H1 Rejected and H0 Accepted.

Test hypotheses:

H0: Returns on stock prices are random.

H1: Returns on stock prices are not random (Confirmation of serial correlation).

3.3.3 Kolmogorov Entropy

This test is based on the characteristic that a random process is not very sensitive to initial conditions. In fact, the Kolmogorov entropy or K entropy test, quantifies the concept of sensitivity to initial conditions. now, consider time series that are infinitely close to each other and, on the other hand, unpredictable to each other. in an chaotic model, as time passes, these two paths diverge and become distinguishable from each other. Kolmogorov entropy [K_2], measures the speed of this event. the new version of this criterion, which was presented in 1983 by Grassberger and Procaccia, as follows:

$$K_2 = \lim_{\varepsilon \to \infty} \lim_{m \to \infty} \lim_{T \to \infty} Ln\ \left[\frac{C^M(\varepsilon)}{C^{M+1}(\varepsilon)}\right] \qquad (5)$$

K2= Kolmogorov Entropy, $C^M\ (\varepsilon)$= Correlation integral with dimension M, Ln= Natural Logarithm, m is taken as the number of nearest degree, ε as the embedding dimension, T= time series.

If a time series is not complicated and be completely predictable: $K_2 \to 0$. if the time series is completely random: $K_2 \to \infty$. In fact, the lesser K_2= the degree of predictability of the system will be higher. in summary, there is the following relationship for chaotic systems: $0 < K_2 < \infty$.

3.3.4 Brock's Test

Brock and Sayers demonstrated in 1988 that if a system exhibits a chaotic process, applying either a linear or a smooth non-linear transformation to the

[DOI: 10.66224/jme.20.2.275]

observations does not affect the obtained correlation dimension. However, if the system does not exhibit a chaotic trend, these transformations will have an impact on the system. Consequently, calculating the correlation once on the original data and then again from the residuals remaining after a linear fit (such as AR) and comparing these two numbers can serve as a test. If these two numbers are similar, it suggests the system is chaotic, whereas if they are different, it indicates the system is non-chaotic.

3.3.5 Correlation Dimension Test

The Correlation Dimension test was introduced by Grassberger and Procaccia in 1983. This algorithm is based on the concept of fractal dimension, a concept that originates from fractal geometry in relation to self-similarity, and has extensive applications in identifying and describing chaotic phenomena. Simulation, in this context, refers to imitation. A model simulator should be able to replicate the structure of a system and generate its behavior. The act of generating behavior does not only mean that the model replicates past observations but also that it can respond to entirely new events and policies. The correlation between variables reflects the system's state in the past. Correlation dimension is a conventional method used to determine the chaos of a system and quantify its chaotic behavior. This quantity will take on an irrational number for chaotic systems. If we consider a sphere with radius R around specific data points, the number of points within this sphere, disregarding its center, can be expressed as follows:

$$C(R) = \frac{1}{N(N-1)} \sum_{i=0}^{(N-dT-1)} \sum_{j=0, j\neq 1}^{(N-dT-1)} \Theta \ \ (R - |x(i)x(j)|) \quad (6)$$

Where N denotes the length of the data domain, and $\Theta(x)$represents the characteristic function.

$$\Theta(\mathrm{x}) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases} \quad (7)$$

When the values of R approach zero, the rate of change of $C(R)$can be expressed as:

$$C(R) = \lim_{R \to 0} k R^{Dc} \quad (8)$$

By solving the above equation, the value of D_C is obtained

[DOI: 10.66224/jme.20.2.275]

$$D_C = \lim_{R \to 0} \frac{\ln C(R)}{\ln R} \tag{9}$$

In this context, D_C represents the correlation dimension. Since the data set is not continuous, the values of R cannot approach zero too closely (no point will be found inside the sphere). To assess the chaotic behavior and estimate the appropriate correlation dimension, one can plot the diagram of $\ln C(R)$ versus $\ln R$. The slope of the linear portion of this diagram provides the value of D_C. If $D_C$is an irrational number, this indicates one of the key criteria for the system being chaotic.

3.3.6 Variance Ratio Test

The "Variance-Ratio" serves to determine whether the variances of two populations from which samples have been drawn are equal or not. The formula for calculating the Variance Ratio Test is as follows: Lo and Mackinlay (1988) introduced the variance ratio test to examine the differences in variances between two populations. The formula for calculating the variance ratio is:

$$\begin{aligned} &variance\ ratio = \frac{k-day\ variance}{k*daily\ variance} - 1 \\ &variance\ ratio \sim N(0, \sigma^2) \\ &\sigma^2 = \frac{2(k-1)(2k-1)}{3kn} \end{aligned} \tag{10}$$

3.3.7 Lyapunov Exponent

The Lyapunov exponent quantifies the rate at which infinitesimally close trajectories diverge over time. Specifically, it measures how rapidly a very small space between two conditions, initially closed, expands over time. The formula utilized for calculating the Lyapunov exponent is as follows:

$$F^t(x_0 + \varepsilon) - F^t(x_0) \approx \varepsilon e^{\lambda t} \tag{11}$$

the left side of this equation is equal to the distance between two initially closed sets, after t step. and the right side of the equation represents the assumption that the distance grows exponentially over time. amount of exponent λ that measured over a long period of time (ideally $t \to \infty$), which is the same Lyapunov exponent. if $\lambda > 0$, then the small distances grow indefinitely over time, which means that the "Stretching Mechanism" is effective. also if $\lambda < 0$, small distances do not grow indefinitely. in other words, system will be gradually deployed in an alternate route.

Indeed, the Lyapunov exponent test is grounded in the characteristic of chaotic series where adjacent points in these series diverge over time. The

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Lyapunov exponent quantifies this divergence through an exponential function. The calculation of the Lyapunov exponent measures the degree of kurtosis in the system's movement. Specifically, it assesses the average speed at which two initially close trajectories exponentially diverge from each other. If the largest calculated Lyapunov exponent yields a positive value, it indicates that the system exhibits chaotic behavior. The calculation method is as follows:

If there is following relationship between $X_n$and X_{n+1}:

$$X_{n+1} = f(X_n) \tag{12}$$

we can represent the distance between X_0 and $X_0 + \varepsilon$ by ε and the distance between $f^n(X_0)$ and $f^n(X_0 + \varepsilon)$ by the exponential function $\varepsilon e^{n\lambda(X_0)}$, in other words:

$$\varepsilon e^{n\lambda(X_0)} = |f^n(X_0 + \varepsilon) - f^n(X_0)| \tag{13}$$

The Lyapunov exponent is a measure that quantifies the rate at which a very small space between two initially close points in a chaotic series diverges over time. Specifically, it assesses the average difference between adjacent points in each iteration, denoted as $\varepsilon e^{n\lambda(X_0)}$, where (λ) is the Lyapunov exponent. This feature of chaotic series, where adjacent points diverge exponentially over time, is the basis for the Lyapunov exponent test. The calculation of the Lyapunov exponent is performed by measuring the amount of kurtosis in the system's movement. Essentially, this method evaluates the average speed at which two initially close trajectories exponentially deviate from each other. If the largest calculated Lyapunov exponent has a positive value, it indicates chaotic behavior. The calculation method involves the following formula:

$$\lambda(x_0) = \lim_{\varepsilon \to 0} \lim_{n \to \infty} \frac{1}{n} \log \left| \frac{f^n(x_0+\varepsilon) - f^n(x_0)}{\varepsilon} \right|$$

or

$$\lambda(x_0) = \lim_{n \to \infty} \frac{1}{n} \log \left| \frac{df^n(x_0)}{dx_0} \right| \tag{15}$$

we can also define the Lyapunov exponent as follows:

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$$\lambda(x_0) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} ln \left| \frac{df(x_i)}{dx_i} \right| \tag{16}$$

The phrase enclosed within the||, represents the average difference between adjacent points in each iteration. The value of the Lyapunov exponent, denoted by (λ), is positive for chaotic series and negative otherwise.

To estimate the Lyapunov exponent, the Jacobian matrix method can be utilized, as presented by Nychka et al. (1992) and detailed in the following section:

The Jacobian matrix is a matrix of partial derivatives, and its determinant, known as the Jacobian, contains all partial derivatives of a vector function. The primary application of the Jacobian is in the transformation of coordinates.

$$\begin{vmatrix} x_t \\ x_{t-L} \\ . \\ . \\ . \\ x_{t-mL+L} \end{vmatrix} = \begin{vmatrix} f(x_{t-L}\,; \ldots ; x_{t-mL}) \\ x_{t-L} \\ . \\ . \\ . \\ x_{t-mL+L} \end{vmatrix} + \begin{vmatrix} e_t \\ 0 \\ . \\ . \\ . \\ 0 \end{vmatrix} \tag{17}$$

where L is the time delay and m is the autoregression length. e_t also represents a sequence of independent random variables with zero mean and constant variance.

$$\begin{gathered} X_t = F(X_{t-L}) + E_t \\ |\, X_M - X'_M| = \left|\, F^M(X_0) - F^M(\, X'_0)\right| \approx \left|(DF^M)_{X_0}(X_0 - X'_0)\right| \\ |\, X_M - X'_M| = |\, T_M(X_0 - X'_0)| \\ \lambda = \lim_{M\to\infty} \frac{1}{2M}\, ln \,|\, lnv_1(M)| \end{gathered} \tag{18}$$

It is important to note that the Lyapunov exponent primarily describes the stretchiness of the system, but stretchiness is not the sole mechanism underlying a chaotic system. Therefore, we conclude that the "folding" mechanism is not accounted for in the Lyapunov exponent.

$$K = \lim_{\varepsilon\to\infty} \lim_{M\to\infty} \lim_{T\to\infty} ln \left[\frac{C^M(\varepsilon)}{C^{M+1}(\varepsilon)}\right] \tag{19}$$

3.3.8 BDS Test

The BDS test, introduced by Brock et al. (1987), is designed to detect whether a random sequence is independently and identically distributed (i.i.d.). This

[DOI: 10.66224/jme.20.2.275]

nonparametric test tests the null hypothesis that the data is i.i.d. against an unspecified alternative, serving as an indirect method to test non-linearity. The BDS test is employed to determine the absence of dependence and to test the residuals obtained from non-linear structures after removing the linear structure from the previously filtered data. It evaluates the time series against the presence of general correlation, thus assessing the potential for a general non-linear process, including chaotic processes, within the observed time series. In fact, the BDS test can serve as a test for nonlinearity or as a test for model mis-specification. The test is formulated as follows:

$$\lim_{T \to \infty} [\, C^M(\varepsilon)\,] = C^1\, (\varepsilon)^M \tag{20}$$

Where $C^M(\varepsilon)$ represents the correlation integral with a dimension of (M). The BDS statistic is based on the standardized difference between these two correlation integrals, which follow a normal asymptotic distribution and are described as follows:

$$W_T^M(\varepsilon) = \{T[C_T^M(\varepsilon) - C_T^1(\varepsilon)/\sigma_T^M(\varepsilon)]\}^{1/2} \tag{21}$$

The methodology for conducting the BDS test involves the following steps:

1) We Extract the linear process of the time series using a model such as ARFIMA.
2) We Calculate the W statistic for the model residuals.
3) If the calculated W statistic is significant, the randomness of the time series is rejected, implying the presence of a nonlinear process in the model.

Brock et al., (1987) demonstrated that the BDS test is more powerful than other tests, provided that the available data are 500 or more, M is 5 or less, and ε is between 0.5 and twice the standard deviation of the data.

3.3.9 Hurst Exponent

The Hurst exponent measures the long-term memory of a time series, reflecting how autocorrelations decrease with increasing lag between values. The R/S method, introduced by Hurst (1951), calculates the ratio between the range of the signal and its standard deviation, offering a key tool for analyzing the persistence in time series.

The test involves considering the time series $X = X_1 \dots X_n$. Initially, the scale of the data is normalized or changed as follows:

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$$Z_r = (X_r - X_m).r = 1 \dots n \tag{22}$$

x_m: series average. in the next step, a new time series is calculated as follows.

$$Y_r = (Z_1 - Z_r).r = 2 \dots n \tag{23}$$

Since the mean of Z is zero, the last value of Y, ie Y_n , will always be zero. The adjusted domain will be equal to:

$$R_n = \max\ (Y_1 \dots Y_n) \text{- } \min\ (Y_1 \dots Y_n) \tag{24}$$

Given that the mean of (Y) is zero, its maximum will always be greater than or equal to zero, and its minimum will always be less than or equal to zero. Consequently, the adjusted domain (R_n) will always be non-negative. Hurst defined the following relationship:

$$(R_{/S})_n = \text{a}.n^H \tag{25}$$

The formula for calculating the Hurst exponent, denoted as (H), involves several key components:

- (R): The rescaled domain, which is the range of the first (n) cumulative deviations from the mean.
- (S): The standard deviation of the time series.
- (a): A constant number.
- (n): The number of observations.
- (H): The Hurst exponent.

$$(R_{/S})_n = \text{log HHHlog(n)} \tag{26}$$

or

$$log(R/S)_n = \log a + H\ log(n) \tag{27}$$

The Hurst exponent coefficient (H) can be estimated through regression analysis. In practice, the Hurst exponent is determined by performing a regression analysis. Based on the results of the Hurst exponent, a value of 0.5 indicates an independent process; a value between 0.5 and 1 suggests a durable time series with extremely long-term memory; and a positive number less than 0.5 indicates the unsustainability of the process.

[DOI: 10.66224/jme.20.2.275]

3.3.10 Artificial Neural Network Test

The Artificial Neural Network (ANN) model serves as a tool for identifying dynamic nonlinear processes, including chaotic processes, within data. These models are capable of estimating and predicting complex nonlinear time series with satisfactory accuracy. An ANN typically consists of three layers: input, intermediate, and output. Input data is connected to the output layer either directly or indirectly through transfer functions in the intermediate layer. The direct relationship of the linear part and the relationship mediated by the intermediate layer define the nonlinear part of the model. One of the notable advantages of ANN is its ability to learn by observing data sets, which allows it to estimate random functions effectively (Chan et al., 2000).

A generalized Artificial Neural Network model (including a linear component) can be represented as follows:

$$y = \beta_0 + x\delta + \sum_i^q G(X\gamma_j)\beta_j + \varepsilon . \quad j = 1. \dots .q \tag{28}$$

$$z = [T^{-\frac{1}{2}} \sum_{t=1}^{T} G_t \, e_t \,]' = \widehat{W}^{-1} \left[T^{-\frac{1}{2}} \sum_{t=1}^{T} G_t \, e_t \,\right]$$

where $\widehat{\mathrm{w}}$ is the compatibility estimator of $\mathrm{W} = \mathrm{var}\left(\mathrm{T}^{-\frac{1}{2}} \sum_{\mathrm{t}=1}^{\mathrm{T}} \mathrm{G_t} \, \mathrm{e_t}\right)$

To mitigate the issue of collinearity between (x) and (G) elements, the primary elements of (G) that are not correlated with (x) can be substituted for (G). In this scenario, an alternative statistic, which is simpler to calculate than the Z statistic, is described below:

$$TR^2 \rightarrow x^2(q) \tag{29}$$

T: Total observations. R^2: The correlation coefficient obtained from the linear regression of (e) residuals is on the main components of G, which are not correlated with x. If this statistic for a time series is greater than the critical values given in the chi-square distribution, it indicates the presence of a dynamic nonlinear process in the data. Otherwise, the data follows a linear process. It is important to note in this test that the rejection of the null hypothesis does not necessarily imply the existence of chaos in the data.

4 Empirical Analysis

In this section, we discuss the potential for predicting stock market trends using various tests and models, with the aim of selecting the most effective test and model for forecasting long-term trends in international stock markets. Our analysis evaluates the predictive power of these methods. Ultimately, we

[DOI: 10.66224/jme.20.2.275]

conduct robustness checks at longer horizons to analyze the non-linear trend and the predictability of market trends under different conditions.

4.1 Descriptive Statistical Results

In Table 3, descriptive indicators such as mean, maximum, minimum, skewness, kurtosis, and standard deviation are presented. The objective of examining research variables descriptively is to understand the distribution and specific characteristics of each variable. The calculated average values of price indices and cash yield over the studied period indicate a bullish trend for the studied indices. However, the calculated standard deviation reveals the dispersion and fluctuations within these indices.

The calculated skewness and kurtosis values suggest that the distribution of the price index and cash yield exhibit greater skewness and kurtosis than a normal distribution. The standard deviation (σ) is a dispersion index that measures how far the data are on average from the mean. A small deviation indicates that the data are close to the mean and exhibit little dispersion, whereas a large deviation signifies significant dispersion. Based on the calculated standard deviation, it can be concluded that the data are close to the average and exhibit little dispersion.

4.2 The Results of The Correlation Dimension Test

Table 4 presents the results of the correlation dimension test. Given that the calculated correlation dimension n (…< n <…) is not within the obtained range, the null hypothesis that stock price returns are random is rejected. This rejection confirms the existence of serial autocorrelation (predictability) in the stock price index.

4.3 The Results of The Hurst Exponent

Table 5 presents the results of the Hurst exponent analysis. Given that the calculated Hurst characteristic falls between 0.5 and 1, it indicates a durable time series with very long-term memory. In other words, it demonstrates the non-random nature of the stock index and the daily predictability of stock prices. Figure 5 illustrates the calculated Hurst exponent for each index.

4.4 The Results of The Lyapunov Exponent

Table 6 presents the results of the Lyapunov exponent analysis. Given that the Lyapunov test results are positive for both the direct method and the Jacobian method, the conclusion that the stock market trend exhibits a chaotic process is supported. Furthermore, due to the high sensitivity of these values to initial

[DOI: 10.66224/jme.20.2.275]

conditions, any chosen initial point will result in paths diverging quickly, with no fixed point or alternating cycle. Figure 7 illustrates the calculated Lyapunov exponent for each index.

4.5 The Results of The Runs Test and Variance Ratio Test

The Runs Test is a statistical procedure used to determine whether a sequence of data within a given distribution has been derived from a random process or not. Table 7 presents the results of the Runs Test. The Variance-Ratio Test is another statistical method that assesses whether the variance of two populations from which samples have been drawn is equal or not. Table 8 displays the results of the Variance-Ratio Test. Additionally, the variance ratio test results for four time series are presented in Figure 6.

Table 3. Results of descriptive statistics

Daily Index Logarithm							Daily Growth of Return						
Mean	σ	Max	Min	Skew	Kurt	Obs	Mean	σ	Max	Min	Skew	Kurt	Obs
4.2529	1.93108	9.590	0.82	0.35099	-0.67	5834	0.00068	0.02895	0.259	-0.26	0.44362	6.60385	5834
16,454	8245.79	37,710	6,547	1.054	-0.21	6542	0.00031	0.01174	0.11365	-0.13	-0.12	11.80	6542
3287.32	615.517	4402.32	1916.26	-0.39	-1.07	3701	0.00022	0.01145	0.0937	-0.14	-0.54	11.1663	3701
72975.9	25561.5	134193.	29435	0.61775	-0.92	4166	0.00043	0.01753	0.1466	-0.14	-0.13	9.09527	4166
2474.22	904.344	6092.05	1011.49	0.49833	0.23239	6465	0.00024	0.01469	0.0986	-0.08	-0.19	5.46719	6465
14842.9	2988.94	22087.2	7566.94	0.47795	-0.23	4314	0.00019	0.01123	0.11957	-0.12	-0.67	18.8445	4314
716.122	507.829	2283.04	169.04	1.19856	0.54425	5943	0.00044	0.01273	0.09962	-0.11	-0.10	4.82381	5943
4701.14	1127.36	7596.91	2403.04	0.46119	-0.49	6625	0.00024	0.01410	0.1118	-0.12	-0.03	5.98984	6625
8560.07	1956.03	13448.0	4802.12	0.16509	-0.67	3599	0.00024	0.01185	0.07392	-0.10	-0.39	4.73287	3599
6522.01	4240.40	16794.4	936	0.63239	-0.68	9272	0.00040	0.01373	0.11402	-0.13	-0.11	6.720214	9272
15447.8	8025.77	33154.1	1894.90	-0.02	-1.11	9389	0.00032	0.01594	0.18824	-0.33	-1.07	32.6912	9389
17528.8	17133.9	72410.3	659.3	1.21798	0.64617	8177	0.00068	0.01598	0.20804	-0.13	0.28327	10.8600	8177
20947.9	3515.30	30426.5	12362.5	0.11225	-0.22	3702	0.00022	0.01519	0.1128	-0.16	-0.64	8.52470	3702
17111.3	5839.97	33753.3	7054.98	0.47266	-0.27	8098	0.00014	0.01451	0.1415	-0.11	-0.05	5.17889	8098
9555.92	1921.53	15945.7	5364.5	0.82646	0.99809	6171	0.00008	0.01424	0.1443	-0.14	-0.08	7.79316	6171
2221.78	348.964	3305.21	1457.64	1.27277	1.05709	3246	0.00016	0.01047	0.0860	-0.08	-0.13	6.41988	3246
548.215	195.516	1045.28	257.56	0.55935	-0.65	3600	0.00041	0.01165	0.0727	-0.11	-0.46	5.35052	3600
6802.24	2700.89	12970.5	1287.59	-0.19	-0.59	8452	0.00030	0.01101	0.1139	-0.09	-0.15	-0.09	8452
941.242	1330.09	8513.54	71.60	3.62646	14.0648	5984	0.00084	0.02047	0.19509	-0.18	0.08286	7.72331	5984
4846.39	1954.75	8014.31	986.90	-0.37	-1.14	10031	0.00025	0.01086	0.09839	-0.12	-0.30	8.72693	10031

Source: Research findings

[DOI: 10.66224/jme.20.2.275]

ID	Country/ Region	Symbol	Definition	Full Period
1	Australia	ASX	Australian Securities Exchange	2000/02/10–2023/21/29
2	America	DJI	Dow Jones Industrial Average	1998/02/01–2023/12/29
3	Belgium	BEL	Bell 20 Index	2009/07/20–2023/12/29
4	Brazil	IBOV	Sao Paulo Stock Exchange Index	2006/09/05–2023/12/28
5	China	SSE	Shanghai Composite Index	1998/05/01–2024/01/05
6	Canada	OSPTX	S&P/TSX Composite index	2006/05/10–2023/12/28
7	Denmark	OMXC20	OMX Copenhagen 20 Index	2000/03/01–2023/12/28
8	France	FCHI	CAC 40	1998/01/02–2023/12/28
9	Finland	OMXHPI	OMX Helsinki All Share Index	2009/07/17–2023/12/28
10	Germany	GDAX	DAX	1988/04/01–2024/01/05
11	Hong Kong	HSI	HANG SENG Index	1987/02/01–2024/01/05
12	India	BSESN	S&P BSE Sensex	1990/02/01–2023/12/29
13	Italy	FTMIB	FTSE MIB	2009/01/06–2023/12/28
14	Japan	NIKKEI	Nikkei 225	1991/04/01–2023/12/29
15	Spain	IBEX	IBEX 35 Index	2000/03/01–2024/01/05
16	South Korea	KOSPI	Korea Composite Stock Price Index	2010/10/11–2023/12/28
17	Sweden	OMXSPI	OMX Stockholm All Share Index	2009/07/17–2023/07/31
18	Swiss	SMI	Swiss Market Index	1991/03/01–2024/01/05
19	TURKEY	BIST 100	Borsa Istanbul stock exchange	2000/04/01–2023/12/28
20	UK	FTSE	FTSE 100	1984/02/04–2023/12/28

[DOI: 10.66224/jme.20.2.275]

Table 4. Results of the correlation dimension test

ID	Country/ Region	Symbol	Full Period	Number Of Positive	Number Of Negative	Number Of Observed	Confidence interval
1	Australia	ASX	2000/02/10–2023/21/29	3037	2797	2685	**(2987/2613<n<3014/2657)**
2	America	DJI	1998/02/01–2023/12/29	3418	3124	3057	**(3389/2989<n<3402/3026)**
3	Belgium	BEL	2009/07/20–2023/12/29	1945	1756	1713	**(1879/1685<n<1910/1703)**
4	Brazil	IBOV	2006/09/05–2023/12/28	2214	1952	1921	**(2174/1857<n<2203/1907)**
5	China	SSE	1998/05/01–2024/01/05	3456	3009	2963	**(3376/2902<n<3410/2924)**
6	Canada	OSPTX	2006/05/10–2023/12/28	2253	2061	1941	**(2189/1893<n<2217/1923)**
7	Denmark	OMXC20	2000/03/01–2023/12/28	3125	2818	2785	**(3072/2714<n<3109/2739)**
8	France	FCHI	1998/01/02–2023/12/28	3451	3174	3054	**(3386/2965<n<3412/3024)**
9	Finland	OMXHPI	2009/07/17–2023/12/28	1897	1702	1657	**(1825/1612<n<1637/2145)**
10	Germany	GDAX	1988/04/01–2024/01/05	4758	4514	4416	**(4709/4389<n<4732/4408)**
11	Hong Kong	HSI	1987/02/01–2024/01/05	4762	4627	4593	**(4712/4518<n<4732/4539)**
12	India	BSESN	1990/02/01–2023/12/29	4132	4045	3957	**(4082/3913<n<4121/3926)**
13	Italy	FTMIB	2009/01/06–2023/12/28	1901	1801	1745	**(1869/1687<n<1896/1723)**
14	Japan	NIKKEI	1991/04/01–2023/12/29	4077	4021	3993	**(4021/3928<n<4034/3956)**
15	Spain	IBEX	2000/03/01–2024/01/05	3125	3046	2978	**(3086/2912<n<3014/2947)**
16	South Korea	KS11	2010/10/11–2023/12/28	1685	1561	1476	**(1617/1411<n<1639/1439)**
17	Sweden	OMXSPI	2009/07/17–2023/07/31	1821	1779	1685	**(1793/1625<n<1802/1643)**
18	Swiss	SMI	1991/03/01–2024/01/05	4313	4139	4124	**(4286/4074<n<4302/4108)**
19	TURKEY	BIST 100	2000/04/01–2023/12/28	3132	2852	2765	**(3068/2719<n<3107/2739)**
20	UK	FTSE	1984/02/04–2023/12/28	5124	4907	4874	**(5069/4814<n<5106/4832)**

Source: Research findings

Table 5. Results of Hurst exponent

ID	Country/ Region	Symbol	Definition	Full Period	Average R/S	log (R/S)	t-stat	p-value	Hurst exponent	Expected Hurst
1	Australia	ASX	Australian Securities Exchange	2000/02/10–2023/21/29	540.3265	5423.3	-0.02	<0.0005	0.52	0.50
2	America	DJI	Dow Jones Industrial Average	1998/02/01–2023/12/29	558.326	6121.3	-0.01	<0.0006	0.51	
3	Belgium	BEL	Bell 20 Index	2009/07/20–2023/12/29	508.2654	4187.3	-0.02	<0.0005	0.57	
4	Brazil	IBOV	Sao Paulo Stock Exchange Index	2006/09/05–2023/12/28	547.3214	6548.3	-0.22	<0.0069	0.56	
5	China	SSE	Shanghai Composite Index	1998/05/01–2024/01/05	556.3698	5789.6	-0.07	<0.0007	0.61	
6	Canada	OSPTX	S&P/TSX Composite index	2006/05/10–2023/12/28	528.4675	5487.7	0.03	<0.0054	0.57	
7	Denmark	OMXC20	OMX Copenhagen 20 Index	2000/03/01–2023/12/28	584.6398	5681.2	-0.24	<0.0000	0.62	
8	France	FCHI	CAC 40	1998/01/02–2023/12/28	546.8247	5317.6	0.05	<0.0005	0.51	
9	Finland	OMXHPI	OMX Helsinki All Share Index	2009/07/17–2023/12/28	567.7448	5874.4	0.04	<0.0023	0.52	
10	Germany	GDAX	DAX	1988/04/01–2024/01/05	577.3698	5446.3	-0.23	<0.0005	0.56	
118	Hong Kong	HSI	HANG SENG Index	1987/02/01–2024/01/05	547.3694	5887.3	-.018	<0.0000	0.58	
12	India	BSESN	S&P BSE Sensex	1990/02/01–2023/12/29	547.3265	5994.3	-0.19	<0.0009	0.52	
13	Italy	FTMIB	FTSE MIB	2009/01/06–2023/12/28	511.3321	6889.3	-0.21	<0.0005	0.64	
14	Japan	NIKKEI	Nikkei 225	1991/04/01–2023/12/29	511.6589	6547.9	-0.27	<0.0000	0.61	
15	Spain	IBEX	IBEX 35 Index	2000/03/01–2024/01/05	569.3214	5787.7	-0.22	<0.0063	0.53	
16	South Korea	KOSPI	Korea Composite Stock Price Index	2010/10/11–2023/12/28	533.6547	5698.9	-0.24	<0.0065	0.56	
17	Sweden	OMXSPI	OMX Stockholm All Share Index	2009/07/17–2023/07/31	502.9541	5445.6	-0.23	<0.0000	0.54	
18	Swiss	SMI	Swiss Market Index	1991/03/01–2024/01/05	583.8341	5237.7	-0.48	<0.0055	0.68	
19	TURKEY	BIST 100	Borsa Istanbul stock exchange	2000/04/01–2023/12/28	568.32	6598.6	0.05	<0.0025	0.51	
20	UK	FTSE	FTSE 100	1984/02/04–2023/12/28	5789.2355	0.5478.	-0.21	<0.0006	0.52	

Source: Research findings

Table 6. Results of largest Lyapunov exponent

ID	Country/ Region	Symbol	Full Period	Direct Method					Method Jacobian				
				Maximum Interruption					Maximum Interruption				
				1	2	3	4	5	1	2	3	4	5
1	Australia	ASX	2000/02/10–2023/21/29	2.264	0.0279	1.854	0.0482	0.345	2.654	4.654	0.0185	0.458	0.765
2	America	DJI	1998/02/01–2023/12/29	2.756	0.0.254	1.324	0.0436	0.287	2.784	4.663	0.0192	0.421	0.714
3	Belgium	BEL	2009/07/20–2023/12/29	3.214	0.0236	2.547	0.0331	0.271	3.217	4.874	0.0262	0.478	0.821
4	Brazil	IBOV	2006/09/05–2023/12/28	2.671	0.1247	2.206	0.2387	0.1756	2.661	5.214	0.0147	0.567	0.796
5	China	SSE	1998/05/01–2024/01/05	1.874	0.3654	1.147	0.4741	0.3821	4.457	4.117	0.0256	0.417	0.725
6	Canada	OSPTX	2006/05/10–2023/12/28	3.287	0.2177	2.547	0.3651	0.2652	3.963	4.997	0.328	0.492	0.784
7	Denmark	OMXC20	2000/03/01–2023/12/28	4.314	0.2359	2.741	0.3744	0.2541	2.477	4.672	0.217	0.589	0.820
8	France	FCHI	1998/01/02–2023/12/28	3.221	0.3327	2.551	0.4482	0.3741	3.124	5.327	0.186	0.623	0.763
9	Finland	OMXHPI	2009/07/17–2023/12/28	2.852	0.2937	1.182	0.3256	0.3127	2.329	4.110	0.396	0.583	0.822
10	Germany	GDAX	1988/04/01–2024/01/05	3.364	0.0347	1.963	0.0459	0.0394	4.459	5.786	0.284	0.407	0.719
11	Hong Kong	HSI	1987/02/01–2024/01/05	4.135	0.3221	2.457	0.4692	0.3451	2.753	5.854	0.377	0.518	0.811
12	India	BSESN	1990/02/01–2023/12/29	2.603	0.1462	1.318	0.2374	0.1762	3.279	4.321	0.289	0.566	0.741
13	Italy	FTMIB	2009/01/06–2023/12/28	4.978	0.2551	1.748	0.3928	0.2691	3.975	5.192	0.341	0.570	0.756
14	Japan	NIKKEI	1991/04/01–2023/12/29	4.221	0.3664	3.499	0.4726	0.3921	2.429	4.735	0.209	0.611	0.863
15	Spain	IBEX	2000/03/01–2024/01/05	2.776	0.0477	1.774	0.0654	0.0542	3.962	5.283	0.290	0.491	0.728
16	South Korea	KOSPI	2010/10/11–2023/12/28	2.869	0.0378	2.307	0.0579	0.0452	4.127	4.556	0.381	0.522	0.624
17	Sweden	OMXSPI	2009/07/17–2023/07/31	3.619	0.3281	2.107	0.4983	0.3716	2.336	5.207	0.247	0.619	0.869
18	Swiss	SMI	1991/03/01–2024/01/05	3.876	0.4217	2.767	0.5691	0.4473	3.227	4.746	0.266	0.573	0.815
19	TURKEY	BIST 100	2000/04/01–2023/12/28	3.942	0.0213	1.348	0.0466	0.0322	4.628	5.459	0.395	0.597	0.728
20	UK	FTSE	1984/02/04–2023/12/28	1.627	0.4125	1.116	0.5736	0.4863	2.118	4.806	0.472	0.617	0.775

Source: Research findings

[DOI: 10.66224/jme.20.2.275]

Table 7. Results of runs test

ID	Country/ Region	Symbol	Full Period	Runs test					
				Expected runs	Total runs	run variance	run stdev	Z-Stat	P-Value
1	Australia	ASX	2000/02/10–2023/21/29	2903.73	2953	1444.02	38.00	1.30	0.0974
2	America	DJI	1998/02/01–2023/12/29	3258.59	3361	1622.11	40.28	2.54	0.0055
3	Belgium	BEL	2009/07/20–2023/12/29	1847.91	1833	921.41	30.35	-0.49	0.6884
4	Brazil	IBOV	2006/09/05–2023/12/28	2081.78	2099	1039.03	32.23	0.53	0.2966
5	China	SSE	1998/05/01–2024/01/05	3218.18	3177	1600.72	40.01	-1.03	0.8484
6	Canada	OSPTX	2006/05/10–2023/12/28	2138.33	2127	1058.67	32.54	-0.35	0.6361
7	Denmark	OMXC20	2000/03/01–2023/12/28	2961.90	2975	1474.92	38.40	0.34	0.3665
8	France	FCHI	1998/01/02–2023/12/28	3265.97	3355	1629.47	40.37	2.21	0.0137
9	Finland	OMXHPI	2009/07/17–2023/12/28	1795.43	1741	894.44	29.91	-1.82	0.9656
10	Germany	GDAX	1988/04/01–2024/01/05	3248.80	3293	1612.37	40.15	1.10	0.1355
11	Hong Kong	HSI	1987/02/01–2024/01/05	3250.75	3144	1614.32	40.18	-2.66	0.9961
12	India	BSESN	1990/02/01–2023/12/29	3261.76	2980	1625.27	40.31	-6.99	1.0000
13	Italy	FTMIB	2009/01/06–2023/12/28	1846.04	1921	919.30	30.32	2.47	0.0067
14	Japan	NIKKEI	1991/04/01–2023/12/29	3270.97	3405	1634.47	40.43	3.32	0.0005
15	Spain	IBEX	2000/03/01–2024/01/05	3076.52	3081	1532.53	39.15	0.11	0.4544
16	South Korea	KOSPI	2010/10/11–2023/12/28	1619.65	1599	806.91	28.41	-0.73	0.7664
17	Sweden	OMXSPI	2009/07/17–2023/07/31	1793.00	1821	891.77	29.86	0.94	0.1742
18	Swiss	SMI	1991/03/01–2024/01/05	3243.71	3184	1607.32	40.09	-1.49	0.9318
19	TURKEY	BIST 100	2000/04/01–2023/12/28	2977.84	2957	1480.62	38.48	-0.54	0.7059
20	UK	FTSE	1984/02/04–2023/12/28	3262.40	3254	1625.91	40.32	-0.21	0.5825

Source: Research findings

Table 8. Results of variance ratio test

ID	SYMBOL	2 Day Return	4 Day Return	8 Day Return	16 Day Return	2 Day Return	4 Day Return	8 Day Return	16 Day Return	2 Day Return	4 Day Return	8 Day Return	16 Day Return
		Variance **VR**	Variance **VR**	Variance **VR**	Variance **VR**	variance of VR **std**	variance of VR **std**	variance of VR **std**	variance of VR **std**	z-stat **p-value**	z-stat **p-value**	z-stat **p-value**	**z-stat** **p-value**
1	Australia	0.0016 **-0.039**	0.0030 **-0.082**	0.00583 **-0.130**	0.01084 **-0.192**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-3.1838 **1.00**	-3.5606 **0.72**	-3.5634 **1.00**	**-3.5247** **0.47**
2	America	0.0002 **-0.096**	0.0004 **-0.133**	0.00088 **-0.195**	0.00171 **-0.222**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-7.7969 **1.00**	-5.7646 **1.00**	-5.343 **1.00**	**-4.0816** **0.45**
3	Belgium	0.0002 **0.036**	0.0005 **0.043**	0.00102 **-0.023**	0.00188 **-0.101**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	2.88693 **0.97**	1.84344 **0.74**	-0.6304 **0.97**	**-1.8589** **0.38**
4	Brazil	0.0006 **-0.854**	0.0011 **-0.865**	0.00210 **-0.875**	0.00421 **-0.875**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-69.104 **1.00**	-37.395 **1.00**	-23.924 **1.00**	**-16.087** **0.41**
5	China	0.0005 **0.013**	0.0010 **0.016**	0.00216 **0.061**	0.00464 **0.138**	0.00015 **0.01236**	0.00054 **0.02313**	0.00134 **0.03657**	0.00296 **0.05442**	1.08514 **0.14**	0.67062 **0.25**	1.6699 **0.05**	**2.53476** **0.35**
6	Canada	0.0002 **-0.861**	0.0004 **-0.866**	0.00088 **-0.875**	0.00172 **-0.879**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-69.642 **1.00**	-37.460 **0.81**	-23.924 **0.74**	**-16.142** **0.38**
7	Denmark	0.0003 **-0.863**	0.0006 **-0.867**	0.00130 **-0.875**	0.00259 **-0.876**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-69.792 **1.00**	-37.470 **0.72**	-23.924 **1.00**	**-16.094** **0.41**
8	France	0.0004 **-0.846**	0.0008 **-0.859**	0.00156 **-0.875**	0.00288 **-0.885**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-68.409 **1.00**	-37.150 **1.00**	-23.924 **1.00**	**-16.263** **0.47**
9	Finland	0.0002 **-0.869**	0.0005 **-0.867**	0.00112 **-0.875**	0.00210 **-0.883**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-70.299 **1.00**	-37.503 **0.68**	-23.924 **1.00**	**-16.221** **0.34**
10	Germany	0.0003 **-0.867**	0.0006 **-0.873**	0.00127 **-0.875**	0.00250 **-0.877**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-70.158 **1.00**	-37.761 **1.00**	-23.924 **1.00**	**-16.119** **0.44**
11	Hong Kong	0.0006 **-0.879**	0.0013 **-0.877**	0.00267 **-0.875**	0.00560 **-0.869**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-71.074 **1.00**	-37.916 **1.00**	-23.924 **1.00**	**-15.968** **0.49**
12	India	0.0007 **-0.886**	0.0015 **-0.880**	0.00321 **-0.875**	0.00709 **-0.862**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.0365724**	0.0029616 **0.0544209**	-71.630 **1.00**	-38.050 **1.00**	-23.924 **0.63**	**-15.839** **0.47**
13	Italy	0.0004 **-0.868**	0.0008 **-0.869**	0.00168 **-0.875**	0.00328 **-0.878**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-70.229 **1.00**	-37.557 **1.00**	-23.924 **1.00**	**-16.133** **0.383**
14	Japan	0.0004 **-0.023**	0.0007 **-0.075**	0.00149 **-0.103**	0.00296 **-0.110**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-1.8770 **1.00**	-3.2273 **1.00**	-2.8064 **0.98**	**-2.0125** **0.47**
15	Spain	0.0004 **-0.852**	0.0008 **-0.862**	0.00157 **-0.875**	0.00296 **-0.882**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-68.877 **1.00**	-37.280 **1.00**	-23.924 **1.00**	**-16.204** **0.48**
16	South Korea	0.0002 **-0.875**	0.0004 **-0.870**	0.00087 **-0.875**	0.00178 **-0.873**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-70.750 **1.00**	-37.596 **1.00**	-23.924 **1.00**	**-16.048** **0.42**
17	Sweden	0.0002 **-0.007**	0.0005 **-0.026**	0.00099 **-0.080**	0.00193 **-0.110**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-0.5330 **0.87**	-1.1151 **0.99**	-2.1987 **0.98**	**-2.0288** **0.48**
18	Swiss	0.0002 **-0.868**	0.0005 **-0.869**	0.00103 **-0.875**	0.00205 **-0.876**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-70.226 **1.00**	-37.555 **1.00**	-23.924 **0.63**	**-16.088** **0.47**
19	TURKEY	0.0010 **0.014**	0.0021 **0.008**	0.00403 **-0.036**	0.00843 **0.009**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	1.12313 **0.64**	0.35533 **0.84**	-0.9803 **0.56**	**0.15716** **0.34**
20	UK	0.0001 **-0.880**	0.0003 **-0.877**	0.00078 **-0.875**	0.00172 **-0.863**	0.000153 **0.012364**	0.000535 **0.02313**	0.001338 **0.036572**	0.0029616 **0.0544209**	-71.215 **1.00**	-37.917 **1.00**	-23.924 **0.63**	**-15.863** **0.36**

Source: Research findings

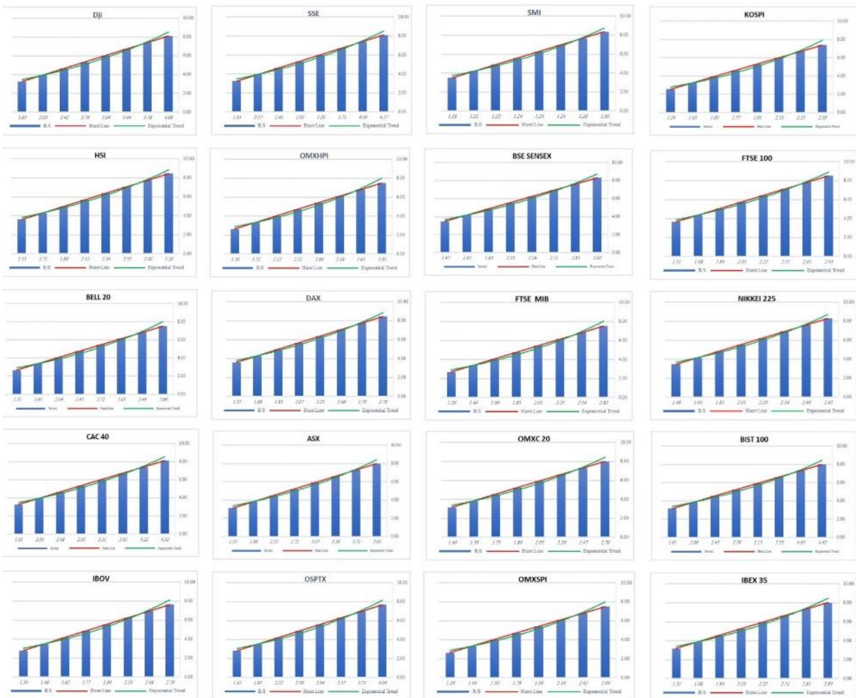

Figure 5. Hurst Exponent Results
Source: Research findings

Figure 6. Variance Ratio Test for Four Time Series
Source: Research findings

[DOI: 10.66224/jme.20.2.275]

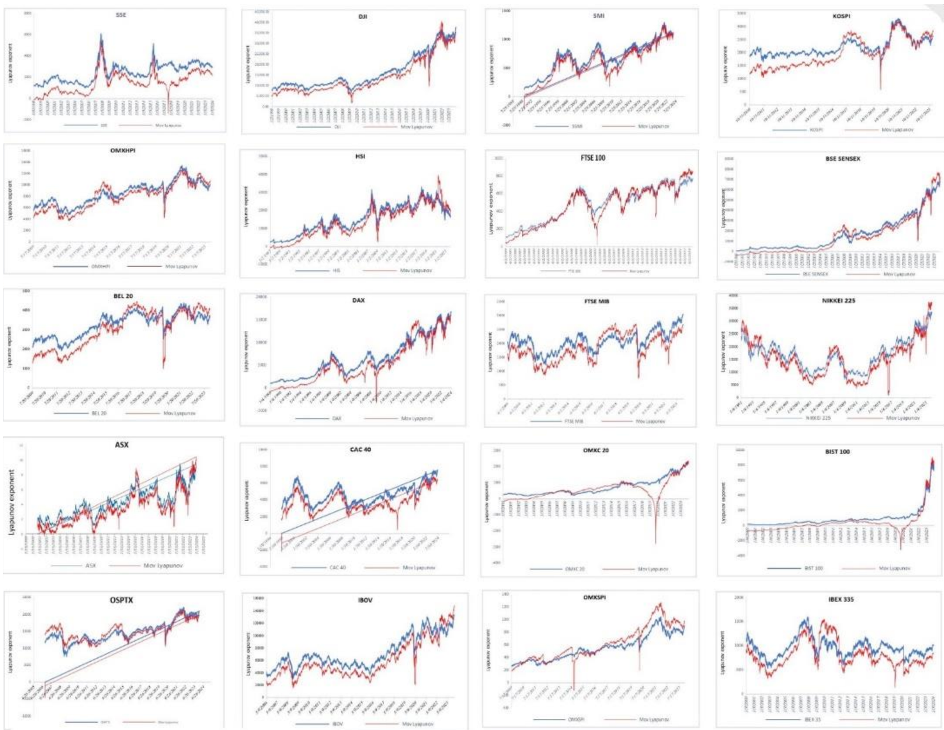

Figure 7. Lyapunov Exponent Results
Source: Research findings

4.6 The Results of BDS Test

The BDS test is designed to detect whether a random sequence is independently and identically distributed (i.i.d.), with the results reported in Table 9. This nonparametric test tests the null hypothesis that the data are i.i.d. against an unspecified alternative, serving as an indirect method to test non-linearity. The BDS test is employed to determine the absence of dependence and to test the residuals obtained from non-linear structures after removing the linear structure from the previously filtered data. It evaluates the time series against the presence of general correlation, thus assessing the potential for a general non-linear process, including chaotic processes, within the observed time series. In fact, the BDS test can serve as a test for nonlinearity or as a test for model mis-specification, as evidenced in studies conducted by Brock et al., 1987.

4.7 The Results of The Unit Root Test

Table 10 illustrates that at significance levels of 1%, 5%, and 10%, the Dickey-Fuller statistic for the growth rate of the stock price index exceeds the critical values in absolute value, indicating statistical significance. Therefore, the null hypothesis regarding the existence of a unit root is rejected. This suggests that the stock price return series is stationary, implying that it does not follow a random walk and shows mean reversion over time.

4.8 The Results of ARFIMA Model

To determine the most suitable model for the growth of the stock price index, the model was fitted with varying intervals, and the results are presented in Table 11, where the optimal ARFIMA model (4, d, 4) and the estimated coefficients are reported at a significance level of 10%. The results indicate that all coefficients are statistically significant, suggesting that the growth of the stock price index in the examined indices exhibits long-term memory, with shocks to these indices taking a significant amount of time to dissipate.

To further investigate, model tests for Goodness-of-fit, normality, and the presence of heteroskedasticity effects were conducted. The test related to the normality of the distribution of observations and residuals of the model revealed that the distribution is not normal. Additionally, the test related to the existence of heterogeneity variance effects in the model demonstrated the presence of heterogeneity variance effects in the residuals of the model, indicating that the model variance should be proportional to the model. Consequently, the model is re-estimated, taking into account the effects of heterogeneity variance.

4.9 The Results of FIGARCH Model

In assessing the stationarity of the model, it is possible to determine whether the growth series of the stock index exhibits stationary or non-stationary behavior. This assessment requires fitting a model such as FIGARCH, which results in autoregressive intervals and moving average, ARCH, and GARCH components. According to Akaike and Schwarz statistics, the best-estimated model is ARFIMA (4, d, 4) – FIGARCH (2, d, 1), as adding other breaks prevents the model from converging. Table 12 presents the results of the FIGARCH model, showing that all coefficients are significant. Since the sum of the positive coefficients is less than one, this indicates the stationarity of the covariance process of the conditional variance. The sum of the coefficients close to one in the model indicates the stability of the shocks and their long-term memory, with the coefficient d also confirming this. Additionally, tests

[DOI: 10.66224/jme.20.2.275]

related to the normality of the distribution of the residuals and the existence of heterogeneity variance effects in the model were reported. The results indicated that the distribution of the residuals was slightly different from the normal distribution, and the residuals of the model did not exhibit autocorrelation. Finally, the test of heterogeneity variance effects indicated that the model does not exhibit heterogeneity variance effects. On the other hand, the forecast made with a one-step-ahead process for ten periods is reported in Table 13.

[DOI: 10.66224/jme.20.2.275]

Table 9. Results of BDS Test

Country/ Region	Dimension 2				3				4				5			
	Z Stat	BDS Stat	SD	Prob	Z Stat	BDS Stat	SD	Prob	Z Stat	BDS Stat	SD	Prob	Z Stat	BDS Stat	SD	Prob
Australia	0.8531	0.016475	0.001654	0.0000	12.453	0.024874	0.002648	0.0000	13.452	0.034687	0.003655	0.0000	14.651	0.045315	0.004282	0.0000
America	0.9587	0.013698	0.001654	0.0000	12.395	0.025647	0.002745	0.0000	13.394	0.035489	0.003751	0.0000	14.514	0.046871	0.004365	0.0000
Belgium	0.9343	0.014657	0.001578	0.0000	12.545	0.023467	0.002564	0.0000	13.564	0.036548	0.003898	0.0000	14.324	0.047853	0.004521	0.0000
Brazil	9.6138	0.014557	0.01654	0.0000	12.604	0.027644	0.002856	0.0000	13.846	0.034127	0.003419	0.0000	14.632	0.045788	0.004366	0.0000
China	0.9347	0.015478	0.01854	0.0000	12.745	0.025117	0.002746	0.0000	13.249	0.036852	0.003755	0.0000	14.570	0.045176	0.004482	0.0000
Canada	0.9465	0.014787	0.001745	0.0000	12.654	0.026597	0.002648	0.0000	13.661	0.035821	0.003547	0.0000	14.795	0.046321	0.004174	0.0000
Denmark	0.9571	0.013654	0.001846	0.0000	12.619	0.025821	0.002745	0.0000	13.124	0.034768	0.003512	0.0000	14.647	0.047651	0.004564	0.0000
France	0.9367	0.014787	0.001746	0.0000	12.307	0.029457	0.002554	0.0000	13.758	0.037964	0.003422	0.0000	14.921	0.048647	0.004287	0.0000
Finland	0.9482	0.015468	0.001921	0.0000	12.578	0.023227	0.002964	0.0000	13.117	0.039642	0.003744	0.0000	14.379	0.049657	0.004233	0.0000
Germany	0.9286	0.013997	0.001964	0.0000	12.739	0.025646	0.002685	0.0000	13.657	0.034379	0.003471	0.0000	14.247	0.047823	0.004489	0.0000
Hong Kong	0.9754	0.017956	0.001671	0.0000	12.657	0.024872	0.002864	0.0000	13.569	0.037654	0.003347	0.0000	14.692	0.048647	0.004396	0.0000
India	0.9346	0.018456	0.001528	0.0000	12.360	0.028787	0.002687	0.0000	13.602	0.038457	0.003645	0.0000	14.330	0.047185	0.004559	0.0000
Italy	0.5943	0.014548	0.001587	0.0000	13.842	0.023664	0.002404	0.0000	13.763	0.038524	0.003878	0.0000	14.286	0.049376	0.004723	0.0000
Japan	0.9610	0.016328	0.001478	0.0000	12.581	0.029657	0.002755	0.0000	13.684	0.036147	0.003628	0.0000	14.552	0.047325	0.004528	0.0000
Spain	0.9566	0.013774	0.001934	0.0000	12.416	0.025831	0.002945	0.0000	13.804	0.039638	0.003843	0.0000	14.750	0.046958	0.004667	0.0000
South Korea	0.9341	0.018644	0.001643	0.0000	12.506	0.234572	0.002864	0.0000	13.724	0.035647	0.003368	0.0000	14.658	0.045871	0.00474	0.0000
Sweden	0.9854	0.016487	0.001756	0.0000	12.496	0.026978	0.002858	0.0000	13.547	0.035597	0.003567	0.0000	14.217	0.048648	0.004673	0.0000
Swiss	0.9547	0.017457	0.001658	0.0000	12.754	0.029527	0.002744	0.0000	13.588	0.039648	0.003245	0.0000	14.598	0.045972	0.004689	0.0000
TURKEY	0.9506	0.013487	0.001478	0.0000	12.478	0.026874	0.002486	0.0000	13.667	0.037284	0.003679	0.0000	14.515	0.047981	0.004322	0.0000
UK	0.9654	0.014678	0.001587	0.0000	12.659	0.025978	0.002597	0.0000	13.429	0.036484	0.003541	0.0000	14.745	0.046925	0.004511	0.0000

Source: Research findings

	ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Table 10. Results of the unit root test

ID	Country/ Region	Symbol	Definition	Full Period	Significance Level			ADF statistic	variable
					%1	%5	%10		
					MacKinnon critical values				
1	Australia	ASX	Australian Securities Exchange	2000/02/10–2023/12/29	-3.8745	-2.8798	-2.5487	-22.1437	The growth rate of the stock price index
2	America	DJI	Dow Jones Industrial Average	1998/02/01–2023/12/29	-3.8745	-2.6478	-2.5987	-24.5478	
3	Belgium	BEL	Bell 20 Index	2009/07/20–2023/12/29	-3.8745	-2.8874	-2.4121	-24.3247	
4	Brazil	IBOV	Sao Paulo Stock Exchange	2006/09/05–2023/12/28	-3.8745	-2.9845	-2.4587	-22.3699	
5	China	SSE	Shanghai Composite Index	1998/05/01–2024/01/05	-3.8745	-2.7847	-2.5693	-23.9887	
6	Canada	OSPTX	S&P/TSX Composite index	2006/05/10–2023/12/28	-3.8771	-2.8236	-2.3791	-23.9781	
7	Denmark	OMXC20	OMX Copenhagen 20 Index	2000/03/01–2023/12/28	-3.8710	-2.6904	-2.4973	-22.9157	
8	France	FCHI	CAC 40	1998/01/02–2023/12/28	-3.8745	-2.9887	-2.3587	-25.1247	
9	Finland	OMXHPI	OMX Helsinki All Share Index	2009/07/17–2023/12/28	-3.8701	-2.7321	-2.5417	-24.2859	
10	Germany	GDAX	DAX	1988/04/01–2024/01/05	-3.8745	-2.8758	-2.4121	-22.5589	
11	Hong Kong	HSI	HANG SENG Index	1987/02/01–2024/01/05	-3.8745	-2.8523	-2.5434	-23.1247	
12	India	BSESN	S&P BSE Sensex	1990/02/01–2023/12/29	-3.8745	-2.6578	-2.5491	-25.3229	
13	Italy	FTMIB	FTSE MIB	2009/01/06–2023/12/28	-3.8745	-2.8741	-2.3626	-23.6512	
14	Japan	NIKKEI	Nikkei 225	1991/04/01–2023/12/29	-3.8745	-2.7898	-2.3479	-22.4751	
15	Spain	IBEX	IBEX 35 Index	2000/03/01–2024/01/05	-3.8745	-2.6587	-2.5823	-23.9823	
16	South Korea	KOSPI	Korea Composite Stock Price	2010/10/11–2023/12/28	-3.8745	-2.9567	-2.4581	-26.3259	
17	Sweden	OMXSPI	OMX Stockholm All Share	2009/07/17–2023/07/31	-3.8745	-2.6547	-2.5482	-25.3247	
18	Swiss	SMI	Swiss Market Index	1991/03/01–2024/01/05	-3.8719	-2.9462	-2.4592	-24.8487	
19	TURKEY	BIST 100	Borsa Istanbul stock exchange	2000/04/01–2023/12/28	-3.8745	-2.8254	-2.4529	-22.2655	
20	UK	FTSE	FTSE 100	1984/02/04–2023/12/28	-3.8745	-2.7845	-2.3697	-23.4577	

Source: Research findings

Table 11 Results of ARFIMA model

ID	Country/ Region	Symbol	FULL PERIOD	d parameter				significance level
				Coefficient	Std. Error	t-value	t-prob	
1	Australia	ASX	2000/02/10–2023/21/29	0.163254	0.08346	1.87	0.074	%10
2	America	DJI	1998/02/01–2023/12/29	0.179854	0.08953	1.94	0.086	
3	Belgium	BEL	2009/07/20–2023/12/29	0.169851	0.08654	1.78	0.071	
4	Brazil	IBOV	2006/09/05–2023/12/28	0.186522	0.08798	1.73	0.079	
5	China	SSE	1998/05/01–2024/01/05	0.168714	0.08635	1.84	0.068	
6	Canada	OSPTX	2006/05/10–2023/12/28	0.176325	0.08547	1.92	0.087	
7	Denmark	OMXC20	2000/03/01–2023/12/28	0.178532	0.08632	1.98	0.091	
8	France	FCHI	1998/01/02–2023/12/28	0.174139	0.08847	1.86	0.064	
9	Finland	OMXHPI	2009/07/17–2023/12/28	0.174128	0.08901	1.79	0.078	
10	Germany	GDAX	1988/04/01–2024/01/05	0.163216	0.08932	1.74	0.073	
11	Hong Kong	HSI	1987/02/01–2024/01/05	0.169635	0.08946	1.87	0.079	
12	India	BSESN	1990/02/01–2023/12/29	0.164785	0.08698	1.96	0.085	
13	Italy	FTMIB	2009/01/06–2023/12/28	0.174159	0.08647	1.79	0.069	
14	Japan	NIKKEI	1991/04/01–2023/12/29	0.169851	0.08759	1.64	0.093	
15	Spain	IBEX	2000/03/01–2024/01/05	0.163239	0.08714	1.74	0.089	
16	South Korea	KOSPI	2010/10/11–2023/12/28	0.166487	0.08896	1.65	0.092	
17	Sweden	OMXSPI	2009/07/17–2023/07/31	0.178594	0.08655	1.61	0.088	
18	Swiss	SMI	1991/03/01–2024/01/05	0.163228	0.08578	1.75	0.097	
19	TURKEY	BIST 100	2000/04/01–2023/12/28	0.177459	0.08761	1.68	0.077	
20	UK	FTSE	1984/02/04–2023/12/28	0.178819	0.088627	1.79	0.083	

Source: Research findings

Table 12. Results of FIGARCH model

d-FIGAR	ARCH(Phi1)				GARCH(Beta1)				GARCH(Beta2)			
t-Prob	Coefficient	Std. Error	t-Value	t-Prob	Coefficient	Std. Error	t-Value	t-Prob	Coefficien	Std. Error	t-Value	t-Prob
0.0187	-0.576457	0.1459	-4.657	0.0000	-0.409541	0.23648	-1.748	0.0645	0.215489	0.12364	1.685	0.0845
0.0196	-0.586592	0.1587	-4.745	0.0000	-0.403584	0.23874	-1.814	0.0692	0.226894	0.12789	1.749	0.0975
0.179	-0.276548	0.1365	-5.321	0.0000	-0.405312	0.22645	-1.754	0.0631	0.217894	0.12998	1.665	0.0824
0.183	-0.568745	0.1348	-4.318	0.0000	-0.407223	0.24887	-1.774	0.0664	0.215486	0.11770	1.709	0.0855
0.0177	-0.579152	0.1487	-5.987	0.0000	0.409661	0.22149	-1.719	0.0632	0.221201	0.11547	1.692	0.0879
0.0182	-0.581023	0.1378	-4.741	0.0000	0.410325	0.22276	-1.825	0.0689	0.210079	0.12644	1.748	0.0724
0.0196	-0.591036	0.1452	-5.756	0.0000	-0.404221	0.24879	-1.776	0.0677	0.223904	0.11704	1.811	0.0733
0.0162	0.571328	0.1358	-4.496	0.0000	0.402648	0.23217	-1.819	0.0646	0.225489	0.11780	1.682	0.0911
0.0185	-0.566471	0.1476	-5.647	0.0000	-0.411852	0.23655	-1.783	0.0697	0.215498	0.12850	1.826	0.0819
0.0172	-0.574697	0.1492	-5.631	0.0000	-0.407544	0.22699	-1.761	0.0709	0.229603	0.11068	1.744	0.0758
0.0189	-0.563987	0.1462	-4.812	0.0000	-0.409661	0.22713	-1.859	0.0682	0.213564	0.12687	1.638	0.0852
0.181	-0.583098	0.1317	-4.846	0.0000	-0.401774	0.23541	-1.879	0.0651	0.216588	0.12703	1.729	0.0825
0.0194	-0.574938	0.1380	-5.434	0.0000	-0.406332	0.23621	-1.794	0.0618	0.224708	0.11739	1.688	0.0839
0.018	-0.572297	0.1494	-4.858	0.0000	-0.406551	0.22664	-1.787	0.0638	0.220014	0.11679	1.841	0.0804
0.0163	-0.564884	0.1528	-5.735	0.0000	-0.409914	0.22856	-1.799	0.0615	0.218002	0.12885	1.690	0.0877
0.0175	-0.573314	0.1549	-4.379	0.0000	-0.413211	0.24879	-1.822	0.0703	0.227604	0.12017	1.678	0.0718
0.0166	-0.591476	0.1543	-4.601	0.0000	-0.401887	0.23669	-1.867	0.0655	0.214973	0.11408	1.806	0.0846
0.0179	-0.563277	0.1593	-4.955	0.0000	-0.414211	0.22587	-1.142	0.0638	0.229632	0.12540	1.772	0.0966
0.0168	-0.578295	0.1537	-5.377	0.0000	-0.408554	0.23648	-1.498	0.0673	0.214794	0.12327	1.719	0.0951
0.0194	-0.5864	0.1482	-4.960	0.0000	-0.4117	0.2487	-1.964	0.0846	0.226486	0.136	1.796	0.0806

Source: Research findings

ID	Country/ Region	SYMBOL	Coefficient	Std. Error	t-Value
1	Australia	ASX	0.432471	0.17584	2.386
2	America	DJI	0.462597	0.18826	2.484
3	Belgium	BEL	0.428796	0.17542	2.413
4	Brazil	IBOV	0.448136	0.17357	2.701
5	China	SSE	0.432187	0.17492	2.457
6	Canada	OSPTX	0.432879	0.17549	2.365
7	Denmark	OMXC20	0.462103	0.18957	2.637
8	France	FCHI	0.421365	0.17416	2.345
9	Finland	OMXHPI	0.453297	0.17169	2.219
10	Germany	GDAX	0.438891	0.18827	2.246
11	Hong Kong	HSI	0.460132	0.17548	2.252
12	India	BSESN	0.432165	0.17313	2.282
13	Italy	FTMIB	0.456527	0.17856	2.467
14	Japan	NIKKEI	0.441268	0.173956	2.377
15	Spain	IBEX	0.439561	0.18965	2.582
16	South Korea	KOSPI	0.427123	0.17563	2.408
17	Sweden	OMXSPI	0.428795	0.17822	2.298
18	Swiss	SMI	0.427119	0.17231	2.376
19	TURKEY	BIST 100	0.436591	0.18549	2.327
20	UK	FTSE	0.4486	0.1746	0.248

4.10 LSTAR Model Results

In certain processes, the assumption of drastic changes around the threshold point may not be reasonable; instead, the rate of adjustment may exhibit a non-linear pattern. In Self-Exciting Models of Smooth Transition (STAR), the self-exciting parameters vary slowly. The model is fitted in two modes: standard mode and transition mode, with the width coefficient from the origin of the model and gamma multiplier fitted for the logistic mode. The results indicate that the coefficients were evaluated based on the prob value; finally, gamma and μ coefficients are also significant. Furthermore, by storing the residual of the model, appropriate tests of the model fit were performed, including a test of the presence of autocorrelation in the model, which indicated that the fitted model does not exhibit autocorrelation at any level. Finally, the STAR model was tested, and the results are presented in Table 14. The test results show the presence of smooth transition effects in the model. Based on H0, which represents the only first-order interdepartmental interaction, the null hypothesis is rejected, and it can be pointed out that there are first- and second-order part interactions in the model. A feature of the LSTAR model is that in this model, the behavior of variables can be modeled symmetrically; therefore, this model can be used in investigating the behavior of variables. The Smooth

Exponential Transition Autoregressive model is defined by the following transfer function.

LSTAR:

$$F(St, \gamma, c) = \frac{1}{1+\exp[-\gamma(s_t - c]} \qquad (30)$$

where F is the transition function, S_t is the transition variable, γ is the slope parameter, and c is the middle parameter between states. In the ESTAR model, whenever γ tends to zero or infinity, this model becomes a linear autoregressive model; because in this case, the transfer function will be constant and otherwise, this model will show nonlinear behavior.

4.11 Results of the ESTAR Model

The ESTAR model is estimated for a variable with a constant number of breaks, which aligns with the LSTAR model in terms of the standard model and transition mode. The width coefficients from the origin of the model and the gamma coefficient for the logistic mode are also considered. The results indicate that the desired coefficients are related to the width from the origin, with gamma being significant as well. Robust statistics of the model fit, including the Durbin-Watson statistic and the coefficient of determination, suggest the absence of autocorrelation and indicate a high explanatory power. Table 15 presents the results of the ESTAR model.

4.12 VIX

Fluctuations in the stock market are a natural consequence of the rise and fall in share prices. Investors utilizing the VIX index can mitigate their investment risk to a certain extent, minimizing potential losses. Consequently, it is evident that an increase in market volatility leads to a rise in the demand for put options, further exacerbating market fluctuations. In such scenarios, the VIX index serves as a valuable tool for evaluating investor fear. It's important for market participants to note that if the VIX index reaches its lowest level, it may signal that the stock market has peaked, while simultaneously, the creation of price bubbles increases the likelihood of a market crash. At this juncture, share prices tend to rise in a bubble, causing shareholders to express significant concern about their investments in the stock market. Therefore, a low VIX index value may indicate the onset of a recession in the stock market. Conversely, a high VIX index value suggests that the market is at a low point, potentially marking the beginning of market growth. Generally, an increase in the VIX index value indicates a rise in future market shareholder fear, leading to a decrease in the stock index, and vice versa. We conclude that the

[DOI: 10.66224/jme.20.2.275]

correlation coefficient between the stock market and the VIX index is negative. Additionally, it's crucial to remember that the VIX index is a critical tool for predicting the state of international markets, enabling market participants to make informed decisions at the right time.

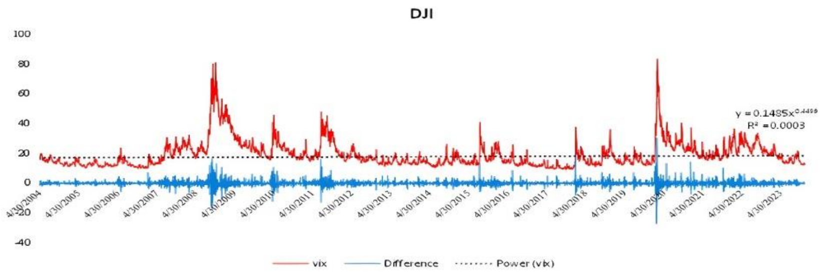

Figure 8. Comparison of Return Differences with the VIX Index
Source: Research findings

In Figure 9, we compare the indicators examined in this study with the VIX index and present the results. Additionally, Table 16, as presented in the appendix at the end of this article, outlines the considered ranges for the VIX index. Conversely, in Figure 8, we compare the return differences with the VIX index.

4.12.1 VIX Limitations

The VIX index is highly sensitive to market conditions and can be influenced by factors such as investor sentiment, geopolitical events, and economic data releases. This sensitivity can lead to sudden spikes or drops in the index, making it challenging to rely on as a consistent measure of volatility. Investors should be cautious when interpreting the VIX index, as it can be prone to short-term fluctuations that may not accurately reflect long-term market trends. In fact, while the VIX is a powerful tool for assessing market volatility, it has its limitations. One of the most significant disadvantages is that it is calculated based on market fluctuations rather than on a base asset. Consequently, verifying and calculating its fair value is not straightforward and presents numerous complications.

4.13 Choosing the Best Model

For evaluating the accuracy and prediction errors of models, criteria such as RMSE, MAE, MSE, and MAPE are commonly used. These metrics measure the difference between actual and predicted values in various ways. They are primarily employed to assess the forecasting power of models on data,

[DOI: 10.66224/jme.20.2.275]

whether within-sample or out-of-sample. Comparing models based on these metrics is logical and common, especially when the goal is to predict future or unknown values. The lower these values, the greater the accuracy of the model in its predictions. Based on the comparison of these statistics, it can be concluded that the FIGARCH model, which captures long-term memory and volatility changes, exhibits superior forecasting power. Moreover, a comparison of the LSTAR and ESTAR nonlinear models shows that there is a statistically significant difference between the two, with the ESTAR model demonstrating stronger predictive ability. Table 15 provides further clarification on this issue.

[DOI: 10.66224/jme.20.2.275]

Table 13. Prediction of ten periods using FIGARCH model

ID	Country/ Region	Symbol	Horizon									
			1	2	3	4	5	6	7	8	9	10
			Mean* / Variance**									
1	Australia	ASX	0.3874* 0.2368**	0.2368* 0.264**	0.2364* 0.2478**	0.1654* 0.259**	0.2347* 0.2424**	0.1698* 0.2297**	0.1487* 0.2132**	0.1602* 0.214**	0.1597* 0.2103**	0.1298* 0.2117
2	America	DJI	0.3984 0.2487**	0.3567 0.381**	0.2864* 0.2952**	0.1846* 0.293**	0.2812* 0.2923**	0.1765* 0.2348**	0.1565* 0.2198**	0.1621* 0.328**	0.1574* 0.2114**	0.1321* 0.2163**
3	Belgium	BEL	0.3265* 0.2354**	0.3256* 0.314**	0.2365* 0.2487**	0.1364* 0.264**	0.2311* 0.2427**	0.1639* 0.2365**	0.1485* 0.2165**	0.1238* 0.267**	0.1218* 0.2124**	0.1178* 0.2145**
4	Brazil	IBOV	0.4612 0.3189**	0.2657 0.264**	0.2354* 0.2456**	0.1784* 0.289**	0.2323* 0.2432**	0.1682* 0.2285**	0.1573* 0.2187**	0.1287* 0.217**	0.1203* 0.2138**	0.1192* 0.2162**
5	China	SSE	0.3748* 0.2147**	0.2698* 0.264**	0.2364* 0.2401**	0.1659* 0.254**	0.2324* 0.2397**	0.1745* 0.2348**	0.1487* 0.2108**	0.1608* 0.227**	0.1516* 0.2100**	0.1238* 0.2103**
6	Canada	OSPTX	0.3998 0.2287**	0.2846* 0.247**	0.2454* 0.2531**	0.1698* 0.267**	0.2434* 0.2512**	0.1678* 0.2381**	0.1423* 0.2186**	0.1613* 0.231**	0.1584* 0.2175**	0.1252* 0.2154**
7	Denmark	OMXC20	0.4611* 0.3885**	0.2874* 0.223**	0.2513* 0.2631**	0.1983* 0.248**	0.2503* 0.2604**	0.1746* 0.2290**	0.1532* 0.2193**	0.1675* 0.206**	0.1528* 0.2184**	0.1374* 0.2182**
8	France	FCHI	0.3445* 0.2113**	0.3165* 0.301**	0.2179* 0.2231**	0.1925* 0.279**	0.2117* 0.2216**	0.1798* 0.2201**	0.1509* 0.2182**	0.1724* 0.279**	0.1691* 0.2173**	0.1328* 0.2153**
9	Finland	OMXHPI	0.3771* 0.2667**	0.2987* 0.268**	0.2137* 0.2289**	0.1637* 0.268**	0.2118* 0.2268*	0.1684* 0.2214*	0.1593* 0.2149**	0.1573* 0.220**	0.1476* 0.2122**	0.1367* 0.2122
10	Germany	GDAX	0.3118* 0.2553**	0.2749* 0.284**	0.2569* 0.2689**	0.1746* 0.217**	0.2513* 0.2651**	0.1745* 0.2364**	0.1570* 0.2197**	0.1681* 0.253**	0.1549* 0.2177**	0.1384* 0.2187
11	Hong Kong	HSI	0.3687* 0.2896**	0.2856* 0.251**	0.2185* 0.2294**	0.1529* 0.201**	0.2131* 0.2257**	0.1637* 0.2216**	0.1407* 0.2109**	0.1512* 0.219**	0.1471* 0.2101**	0.1164* 0.2104**
12	India	BSESN	0.4153* 0.3281**	0.3417* 0.321**	0.2563* 0.2691**	0.1743* 0.244**	0.2542* 0.2633**	0.1678* 0.2308**	0.1523* 0.2188**	0.1632* 0.284**	0.1566* 0.2165**	0.1218* 0.2173**
13	Italy	FTMIB	0.3255* 0.2369**	0.3617* 0.328**	0.2497* 0.2537**	0.1795* 0.278**	0.2442* 0.2501**	0.1768* 0.2274**	0.1544* 0.2175**	0.1679* 0.293**	0.1627* 0.2162**	0.1354* 0.2168**
14	Japan	NIKKEI	0.3993 0.2341**	0.2967* 0.274**	0.2687* 0.2793**	0.1634* 0.270**	0.2622* 0.2709**	0.1639* 0.2205**	0.1527* 0.2162**	0.1568* 0.259**	0.1495* 0.2147**	0.1276* 0.2144**
15	Spain	IBEX	0.4189* 0.3263**	0.2693* 0.268**	0.2391* 0.2489**	0.1669* 0.234**	0.2312* 0.2419**	0.1738* 0.2388**	0.1584* 0.2177**	0.1657* 0.233**	0.1559* 0.2156	0.1253* 0.2135**
16	South Korea	KOSPI	0.3964* 0.2891**	0.2795* 0.275**	0.2146* 0.2285**	0.1585* 0.266**	0.2124* 0.2216**	0.1722* 0.2203**	0.1473* 0.2103**	0.1409* 0.263**	0.1378* 0.2101**	0.1182* 0.2100**
17	Sweden	OMXSPI	0.3776 0.2638**	0.2846* 0.268**	0.2474* 0.2582**	0.1938* 0.229**	0.2421* 0.2518**	0.1607* 0.2219**	0.1593* 0.2133**	0.1542* 0.228**	0.1467* 0.2108**	0.1249* 0.2119**
18	Swiss	SMI	0.4132* 0.3162**	0.3964* 0.349**	0.2784* 0.2897**	0.1638* 0.281**	0.2709* 0.2807**	0.1626* 0.2383**	0.1557* 0.2149**	0.1544* 0.314**	0.1520* 0.2134**	0.1360* 0.2137**
19	TURKEY	BIST 100	0.3572* 0.2359**	0.2761* 0.249**	0.2639* 0.2764**	0.1797* 0.246**	0.2601* 0.2728**	0.1657* 0.2294**	0.1488* 0.2170**	0.1697* 0.211**	0.1619* 0.2128**	0.1527* 0.2159**
20	UK	FTSE	0.3946* 0.3487**	0.2951* 0.267**	0.2504* 0.2628**	0.1609* 0.237**	0.2765* 0.2846**	0.1746* 0.2341**	0.1563* 0.2106**	0.1478* 0.246**	0.1345* 0.2190**	0.1189* 0.2106**

Source: Research findings

Table 14. Results of STAR model

ID	Country/ Region	Symbol	FULL PERIOD	TEST				
				Linearity	H01	H02	H03	H12
				F-stat → Sig				
1	Australia	ASX	2000/02/10–2023/21/29	9.123265 → **0.0000**	5.659874 → **0.0174**	13.259874 → **0.0003**	5.325649→ **0.0145**	**10.321459** → 0.0000
2	America	DJI	1998/02/01–2023/12/29	9.886592 → **0.0000**	6.369874 → **0.0284**	17.632587 → **0.0001**	7.325513 → **0.0156**	10569874 → **0.0000**
3	Belgium	BEL	2009/07/20–2023/12/29	9.478836 → **0.0000**	5.632147 → **0.0169**	14.659877 → **0.0006**	6.213564 → **0.0165**	10.325896 → **0.0000**
4	Brazil	IBOV	2006/09/05–2023/12/28	8121478 → **0.0000**	5.187741→ **0.0195**	15.256454→ **0.0012**	5.369965→ **0.0184**	10.327412 → **0.0000**
5	China	SSE	1998/05/01–2024/01/05	11.326588 → **0.0000**	5.632147 → **0.0254**	16.325682 → **0.0007**	5.321475 → **0.0162**	11.021587 → **0.0000**
6	Canada	OSPTX	2006/05/10–2023/12/28	9.654412 → **0.0000**	6.789632 → **0.0172**	15.879632→ **0.0001**	5.289743 → **0.0146**	10.289746 → **0.0000**
7	Denmark	OMXC20	2000/03/01–2023/12/28	10.987463 → **0.0000**	7.685887 → **0.0145**	15.369951 → **0.0009**	6.324577 → **0.0195**	10.349963 → **0.0000**
8	France	FCHI	1998/01/02–2023/12/28	9.558991 → **0.0000**	5.582147 → **0.0169**	15.148563 → **0.0005**	5.447132→ **0.0147**	10.915735→ **0.0000**
9	Finland	OMXHPI	2009/07/17–2023/12/28	9.787479 → **0.0000**	7.568971 → **0.0261**	16.364127→ **0.0017**	6.321447 → **0.0191**	11.297493 → **0.0000**
10	Germany	GDAX	1988/04/01–2024/01/05	9.827196 → **0.0000**	6.122147 → **0.0181**	15.369957 → **0.0015**	5.699879→ **0.0000**	10.163489→ **0.0000**
11	Hong Kong	HSI	1987/02/01–2024/01/05	9.362978→ **0.0000**	6.417896 → **0.0172**	15.874123→ **0.0021**	5.654711→ **0.0194**	11.287631 → **0.0000**
12	India	BSESN	1990/02/01–2023/12/29	9.362978→ **0.0000**	5.987456 → **0.0235**	15.217485→ **0.0027**	9.123265 → **0.0152**	10.214784 → **0.0000**
13	Italy	FTMIB	2009/01/06–2023/12/28	9.215468→ **0.0000**	7.124589 → **0.0169**	14.664712 → **0.0018**	5.326588 → **0.0138**	11.214281→ **0.0000**
14	Japan	NIKKEI	1991/04/01–2023/12/29	9.325581→ **0.0000**	5.558963 → **0.0264**	15.321478→ **0.0031**	6.125478 → **0.0183**	10.326597 → **0.0000**
15	Spain	IBEX	2000/03/01–2024/01/05	11.326577 → **0.0000**	6.215477→ **0.0191**	17.364892 → **0.0025**	7.122369 → **0.0163**	11.879652 → **0.0000**
16	South Korea	KOSPI	2010/10/11–2023/12/28	11.326574 → **0.0000**	5.988521 → **0.0174**	14.231554→ **0.0024**	5.632147 → **0.1034**	11.869987 → **0.0000**
17	Sweden	OMXSPI	2009/07/17–2023/07/31	9.791456 → **0.0000**	5.214789 → **0.0214**	15.963649 → **0.0035**	5.321478 → **0.0156**	10.164788 → **0.0000**
18	Swiss	SMI	1991/03/01–2024/01/05	10.258964 → **0.0000**	6.214789 → **0.0185**	17.321474 → **0.0011**	7.214789 → **0.0163**	10.326598→ **0.0000**
19	TURKEY	BIST 100	2000/04/01–2023/12/28	9.322459 → **0.0000**	6.214563 → **0.0228**	16.328974 → **0.0001**	6.336987 → **0.0185**	10.155867→ **0.0000**
20	UK	FTSE	1984/02/04–2023/12/28	9.648597 → **0.0000**	5.443964 → **0.0228**	14.357892 → **0.0001**	5.851433 → **0.0185**	10.36741→ **0.0000**

Source: Research findings

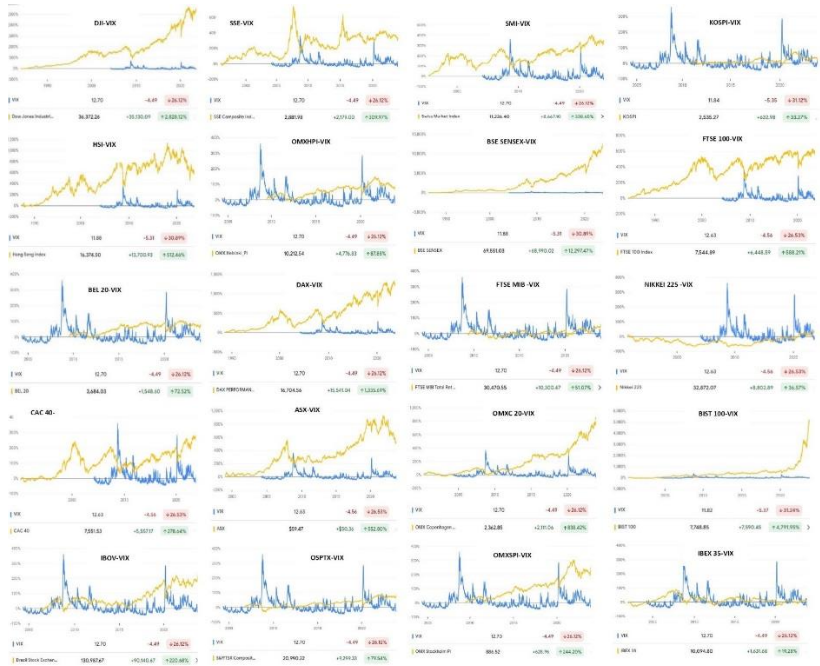

Figure 9. Examining the Time Series of the VIX and Studied Indices
Source: Research calculation

[DOI: 10.66224/jme.20.2.275]

Table 15. Results of evaluating the models' predictive ability

ID	Country/ Region	Symbol		ARFIMA				FIGARCH				LSTAR				ESTAR			
				MSE	MAE	RMSE	MAPE	MSE	MAE	RMSE	MAPE	MSE	MAE	RMSE	MAPE	MSE	MAE	RMSE	MAPE
1	Australia	ASX	2000/02/10–2023/21/29	0.1065	0.2952	0.345	0.1587	0.0532	0.1749	0.224	0.1254	0.0728	0.2254	0.2565	0.0874	0.0627	0.1847	0.2147	0.091
2	America	DJI	1998/02/01–2023/12/29	0.1044	0.2917	0.345	0.1596	0.0575	0.1678	0.2152	0.1274	0.0741	0.2241	0.2598	0.0881	0.0656	0.1962	0.2189	0.094
3	Belgium	BEL	2009/07/20–2023/12/29	0.1098	0.2874	0.336	0.1574	0.0554	0.1651	0.2001	0.1263	0.0721	0.2365	0.2578	0.0827	0.0651	0.1854	0.2163	0.082
4	Brazil	IBOV	2006/09/05–2023/12/28	0.1082	0.2941	0.347	0.1602	0.0554	0.1859	0.2244	0.1325	0.0741	0.2259	0.2569	0.0885	0.0652	0.1954	0.2109	0.084
5	China	SSE	1998/05/01–2024/01/05	0.1028	0.2839	0.345	0.1541	0.0522	0.1779	0.2195	0.1287	0.0744	0.2392	0.2547	0.0879	0.0637	0.1985	0.2071	0.097
6	Canada	OSPTX	2006/05/10–2023/12/28	0.1096	0.2863	0.3418	0.1584	0.0617	0.1688	0.2064	0.1367	0.0728	0.2367	0.2507	0.07177	0.0693	0.1752	0.2156	0.096
7	Denmark	OMXC20	2000/03/01–2023/12/28	0.1043	0.2854	0.3591	0.1643	0.0567	0.1696	0.2007	0.1316	0.0746	0.2241	0.2522	0.0782	0.0674	0.1638	0.2365	0.090
8	France	FCHI	1998/01/02–2023/12/28	0.1063	0.2859	0.339	0.1563	0.0541	0.1685	0.2141	0.1374	0.0785	0.2282	0.2621	0.0788	0.0611	0.1967	0.2195	0.093
9	Finland	OMXHPI	2009/07/17–2023/12/28	0.1059	0.2904	0.3627	0.1632	0.0634	0.1719	0.2177	0.1342	0.0792	0.2609	0.2569	0.0802	0.0649	0.1706	0.2426	0.084
10	Germany	GDAX	1988/04/01–2024/01/05	0.1074	0.2956	0.341	0.1564	0.0755	0.1671	0.2121	0.1274	0.0745	0.2241	0.2596	0.0847	0.0647	0.1658	0.2217	0.081
11	Hong Kong	HSI	1987/02/01–2024/01/05	0.1046	0.2895	0.394	0.1574	0.0696	0.1647	0.2165	0.1254	0.0747	0.2312	0.2541	0.0876	0.0628	0.1745	0.2153	0.089
12	India	BSESN	1990/02/01–2023/12/29	0.1017	0.2968	0.386	0.1632	0.0565	0.1621	0.2043	0.1384	0.0736	0.2269	0.2574	0.0852	0.0654	0.1937	0.2133	0.082
13	Italy	FTMIB	2009/01/06–2023/12/28	0.1069	0.2915	0.362	0.1577	0.0547	0.1553	0.2046	0.1272	0.0782	0.2247	0.2563	0.0864	0.0628	0.1647	0.2109	0.087
14	Japan	NIKKEI	1991/04/01–2023/12/29	0.1021	0.2965	0.341	0.1654	0.0547	0.1786	0.2057	0.1269	0.0752	0.2259	0.02639	0.0827	0.0659	0.1945	0.2145	0.089
15	Spain	IBEX	2000/03/01–2024/01/05	0.1088	0.2892	0.329	0.1563	0.0547	0.1684	0.2176	0.1287	0.0767	0.2387	0.2587	0.0759	0.0694	0.1854	0.2102	0.084
16	South Korea	KOSPI	2010/10/11–2023/12/28	0.1096	0.2963	0.389	0.1593	0.0554	0.1673	0.2046	0.1227	0.0742	0.02361	0.2684	0.0863	0.0623	0.1981	0.2156	0.074
17	Sweden	OMXSPI	2009/07/17–2023/07/31	0.1089	0.2969	0.348	0.1578	0.0552	0.1685	0.2102	0.1324	0.0746	0.2247	0.2582	0.0882	0.0648	0.1688	0.2163	0.094
18	Swiss	SMI	1991/03/01–2024/01/05	0.1006	0.2821	0.3744	0.1544	0.0511	0.1673	0.2159	0.1396	0.0760	0.2392	0.2555	0.0806	0.0645	0.1746	0.2416	0.064
19	TURKEY	BIST 100	2000/04/01–2023/12/28	0.1041	0.2947	0.395	0.1594	0.0963	0.2514	0.2841	0.1291	0.0749	0.2374	0.2629	0.1155	0.0677	0.1932	0.2147	0.079
20	UK	FTSE	1984/02/04–2023/12/28	0.1048	0.2911	0.335	0.1557	0.0912	0.2456	0.2893	0.1259	0.0795	0.2287	0.2576	0.1158	0.0647	0.1769	0.2178	0.098

Source: Research findings

5 Discussion and Conclusion

The advent of significant advancements in computational tools over the past few decades has enabled the application of theories predicated on the existence of specific nonlinear patterns or seemingly random chaos. The application of chaos theory within the financial markets, particularly in the stock market as highlighted in this study, posits that prices in these markets are governed by a certain non-linear relationship, which can be predicted with accuracy if the initial conditions are known. This research employed various tests to investigate the chaotic process of the stock price index's time series.

Each test introduced has its own set of limitations and advantages, and by conducting a selection of them, we have gained a deeper understanding of the conditions that govern the data, as well as the predictability of market movement trends. Indeed, chaos theory, a complex mathematical theory, seeks to elucidate the influence of seemingly insignificant but undeniable factors on chaotic or random events that ultimately impact investor decision-making, stock price forecasting, and market movement trends. Accepting the chaotic process of the stock market is equivalent to acknowledging its inefficiency, at least at the first level of efficiency. Despite the undeniable evidence of the non-linear nature of the price index in the stock market, there remains a considerable distance to definitively prove the existence of this property.

Given the limitations and advantages of each test mentioned in this article, we conducted various tests to better understand the data. The findings indicate that the stock price index, investigated from January 1984 to January 2024, exhibits a dynamic non-linear process with short-term predictability. Consequently, based on the investigations and confirmation of the hypotheses that the stock price index in the investigated markets is non-random and possesses a non-linear structure, and the existence of chaos is confirmed, the results of the tests presented in Table 15 demonstrate the predictive power of the models. Specifically, for long-term forecasting, the FIGARCH and ESTAR models, as shown in Table 15, are found to have higher reliability.

In comparison to previous studies, the results of this research align with the growing body of work suggesting that the prices of financial market indices are governed by non-linear processes and chaotic dynamics. However, while the evidence for the non-linear nature of the price index in the investigated financial markets is undeniable, this study further supports previous research that confirms the chaotic behavior in financial markets. The use of advanced forecasting models such as FIGARCH and ESTAR provides new insights into the short-term predictability of financial market indices, which were examined in prior studies with varying degrees of accuracy and

[DOI: 10.66224/jme.20.2.275]

predictive power. Unlike traditional linear models, which are limited in capturing the complexities of financial markets, this research confirms that incorporating non-linear methods offers a more robust framework for forecasting market behavior. While some studies have pointed to the inefficiency and random-like nature of financial market indices, this research enhances our understanding by providing empirical evidence of the non-random, non-linear structure of market index movements. Therefore, the findings contribute to a more nuanced perspective on financial market dynamics, offering implications for investors who seek to develop strategies that account for the inherent unpredictability and chaos of financial market indices.

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Appendix 1

In Table 16, the different ranges of the VIX index and investors' perspectives on future risks are analyzed. The table divides the VIX index into three categories: "less than 12," "between 12 and 20," and "above 20." For each row, investors' and shareholders' views on the future and their assessment of risks, ranging from "low risk" to "high risk," are examined.

[DOI: 10.66224/jme.20.2.275]

Table 16
VIX index ranges

Ranges / Row	Less than 12	Between 12 and 20	Above 20
1	Low risk	Balanced risk from the future	high risk
Investors' view on the future			
2	Shareholders are optimistic about the future	Balanced risk from the future	Shareholders are not optimistic about the future

Source: Research findings

Appendix 2

Table 17 provides the abbreviations and definitions of key terms related to the study, including the volatility index, stock market, total index, stock return, price of stock index, and volume of stock index.

Table 17
Abbreviations

VIX	Volatility Index
SM	Stock Market
TI	Total Index
SR	Stock Return
PI	Price of Stock Index
Vol	Volume of Stock Index

Source: Research findings

[DOI: 10.66224/jme.20.2.275]