

## Original Research Article

# Proposing an Innovative Model Based on the Sierpinski Triangle for Forecasting EUR/USD Direction Changes

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The Sierpinski triangle is a fractal that is commonly used due to some of its characteristics and features. The Forex financial market is among the places wherein this triangle's characteristics are effective in forecasting the prices and their direction changes for the selection of the proper trading strategy and risk reduction. This study presents a novel approach to the Sierpinski triangle and introduces an innovative model based on it to forecast the direction changes in currency pairs, particularly EUR/USD. The model proposed in this study is dependent on the number of data selected for forecasting. The number of data is, in fact, the area of the initial triangle and the forecasted value of the self-similar triangles formed in each stage. For the performance assessment of the proposed method within one year (03/01/2019 to 28/02/2020), daily EUR/USD closed price data was classified into three categories, namely the training (70%), testing (20%), and validation (10%). Three approaches were proposed that led to forecasting the mean direction accuracy and the best result of over 60 percent in the third approach and over 50 percent in the first and second approaches. Results reflect the satisfactory improvements in the third approach compared to the econometrics, time-series, and machine learning methods. Moreover, the optimal number of data for the model is selected such that the difference between the accuracy of the direction forecasting in the training category and testing category is above 0.6 and below 0.05.

**Keywords:** Fractal, Sierpinski Triangle, Forex Financial Market, Direction Changes, EUR/USD

**JEL Classification:** C69, C65, G10, G17, F31

## 1 Introduction

The foreign exchange (FX) market is one of the most important investment markets, and exchange rate forecasting has always been among the most

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challenging research topics. The foreign exchange market (Forex market) is a place where traders analyze and forecast with two technical (retrospective analysis) and fundamental approaches (analysis of the determining variables) and then exchange different currencies via the Internet by placing purchase and sales orders and closing the orders. If a trader's analysis of an increase in the exchange rate reveals a positive change in the direction and an ascending trend, they place a purchase order while the sales order is the opposite of the purchase order. Besides, a trader can also close an order they opened depending on their analysis, the achievement of the desired profit, or fear of losing the trade. In each period, there are four main open exchange rates (the exchange rate at the beginning of the period), the highest rate (the maximum exchange rate within the period), the lowest rate (the minimum exchange rate within the period), and closed rate (the exchange rate at the end of the period), which can be considered on a daily, weekly, etc. basis. Pip<sup>1</sup> is the measure and the smallest unit for the change in the ratio of a currency pair, which equals 0.0001 of the currency pair rate (except for JPY that equals 0.001), and the profit and loss changes are communicated based on this ratio (Haeri, Hatefi, & Rezaie, 2015; Haeri, Rabbani, & Habibnia, 2011).

Moreover, making FX risk management decisions is important, and the accurate forecasting of the direction and trend of the exchange rate, as the fundamental component of every effective trading strategy (Menkhoff & Taylor, 2007), is vital to risk management (Lee & Wong, 2007). The different econometric and time series models that have been proposed have staged a weak performance in forecasting the currency direction changes, while the performance of the machine learning models is contingent on the features of the input data provided to train the models despite their better (Galeshchuk & Mukherjee, 2017). Therefore, proposing a model that leads to better results than other models and is not influenced by the input features is substantially important.

The present study is an attempt to forecast the daily Euro-United States Dollar (EUR/USD) exchange changes about the fractal theory and with a new approach to the Sierpinski triangle and its properties to be able to develop a proper instrument for traders in the Forex market for the correct management of risks and achievement of the expected profit.

The term "fractal" was coined by Mandelbrot (1975, 1984) to describe complex geometric objects with a high degree of self-similarity (Mandelbrot, 1975, 1984). Self-similarity suggests it repeats its fractal image on each level

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<sup>1</sup> Price interest point

(Bhutta, 1999). In general, a fractal, as a geometric shape, has the following properties.

- 1) It has self-similarity.
- 2) It is very complex on the micro-scale.
- 3) It is not followed by an integer (e.g., 1.5) (Baas, 2002; Bhutta, 1999).

The Sierpinski triangle is a fractal introduced by Waclaw Sierpinski in 1916, and it was developed using a recursive algorithm (Jurgens, Peitgen, & Saupe, 1978; Sierpinski, 1915). This triangle properly mirrors fractals' properties, which are triangles in smaller triangles, and so on (Bhutta, 1999).

Based on the discussion above, this study attempts to find the answers to the following questions and effectively assess the model.

- 1) How much historical data does lead to the best accuracy in the training, testing, and validation categories in the forecasting process? (The best value for variable  $P$ , which is described in the following.)
- 2) To what extent can the proposed model accurately forecast the daily EUR/USD exchange rate changes within one year? (Does the model have acceptable accuracy?)
- 3) Under what circumstances does the proposed model lead to better results than the methods proposed so far?

In this paper, a brief literature review of fractals and the Sierpinski triangle and direction forecasting in the Forex market is provided in section 2, and the complete description of the Sierpinski triangle and the proposed method with the description of the research data is followed in section 3. Thereafter, in section 4, the accuracy of the proposed method in forecasting the changes in the daily EUR/USD exchange rate is validated using the training, testing, and validation data categories to approve the model's capability, and Section 5 presents the conclusions and future research.

## 2 Literature Review

In this section, the previous studies on the application of fractals and the Sierpinski triangle in different areas, especially the financial market, and the methods used for forecasting the currency rate changes are studied and analyzed.

### 2.1 Fractals and Sierpinski Triangle

The fractal theory is a data mining-based technique for detecting patterns and has been utilized in various areas such as road safety analysis (Haleem, Alluri, & Gan, 2016), geomorphology (Baas, 2002), corrosion forecasting models (Li, Yongji, & Shujian, 2006), sensitivity analysis of wind speed parameters

(Nikolić et al., 2017), permeability forecasting, energy use forecasting (Yu et al., 2020) and other areas.

The fractal theory and the Sierpinski triangle have been used in financial areas such as the Forex market and the stock market. Stock market forecasting One of their successful applications is (Haleem, Alluri, & Gan, 2016). The Sierpinski fractal sale network and penetration system have been used to develop an agent-based financial price model (Pei, Wang, & Fang, 2017) and analyze the statistical behaviors of the price change volatility (Dong & Wang, 2014) in financial markets. The hybrid models and fractal analysis are also used to forecast the near values in the Warsaw Stock Exchange (Paluch & Jackowska-Strumiłło, 2014, 2018). Modeling the financial dynamics in Sierpinski gasket to regenerate financial markets' statistical features (Xing & Wang, 2019), developing a stock price model, and detecting the stock market complexities (Zhang & Wang, 2019) constitute a new approach to the construction of a financial micro-mechanism. From the graphical point of view, the Sierpinski gasket is similar to the fractal theory, and it complies with the well-known Sierpinski fractal gasket (Xing & Wang, 2019). The other applications include the use of fractal models for the estimation of news and the impact on the market volatility (Romanov et al., 2007) and stock market forecasting modeling to establish a new border in the areas related to important decision-making in nonlinear, dynamic, and volatile environments (Karaca, Zhang, & Muhammad, 2020).

## 2.2 Direction Forecasting in the Forex Market

Many studies have used various methods to forecast the Forex market orientation. For instance, a combination of the artificial neural network (ANN) and fuzzy logic inference (FLI) is used in Lee & Wong's (2007) paper. In Peramunetilleke & Wong (2002), the exchange rate (ascending, descending, and fixed) is forecasted using the news headlines. In El Shazly & El Shazly (1997), the three currencies, namely the British Pound (GBP), Deutsche Mark (DME), and the Japanese Yen (JPY), are forecasted using a neural network model (Multilayer perceptron (MLP)). This model is used to assess accurate direction forecasting. In Haeri, Hatefi, & Rezaie (2015), the direction (increase or decrease) of the daily EUR/JPY exchange rate is forecasted with a hybrid approach using the hidden Markov models and CART<sup>1</sup> classification algorithms. In Plakandaras et al. (2015), investors' feelings are taken into account to forecast the exchange rate orientation. In Galeshchuk & Mukherjee

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<sup>1</sup> Classification And Regression Trees

(2017), deep convolution neural networks (CNN) are used to forecast the direction of changes in USD/EUR, USD/GBP, and USD/JPY, which led to better results than the other econometric, time-series, ANN, and deep neural network methods. In Özorhan, Toroslu, & Şehitoğlu (2017), support vector machines (SVM) and genetic algorithms (GA) are used along with an axial force forecasting model to forecast the direction and movement of currency pairs in the Forex market, while in (Moews, Herrmann, & Ibikunle, 2019) deep neural networks are utilized to forecast the direction trend changes in the financial time series based on correlations. In (Bakhach, Tsang, & Jalalian, 2016), the trend direction forecasting problem in the Forex market is solved based on the framework of direction changes for EUR/CHF<sup>1</sup>, GBP/CHF, and USD/JPY as an approach for summarizing market prices except for the time series. Exchange rate forecasting through the general spectrum series and nonlinear models, its assessment with the mean direction forecasting (Hong & Lee, 2003), and development of mathematical models for decision making and forecasting of the exchange rate direction (increase or decrease) within a particular period based on the representative values during a EUR/JPY period in Forex (Haeri, Rabbani, & Habibnia, 2011) have been among the subjects of other studies. In Lai (1990), a non-parametric test is used to assess the forecasts of the money market services' survey, and it is used for the forecasting of the currency change orientation. Table 1 shows a more detailed description of the reviewed works.

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<sup>1</sup> Swiss Franc

Table 1

*A more detailed description of reviewed works*

Authors (Year)	Important variables	Predicted variables	Method	Data	Sample size	Result
Lee (2007)	Interest rates, crude oil prices & currency order flow	AUD <sup>1</sup> / USD exchange rates	ANN & FLI	Daily data for 1991- 2000	2194	Prediction accuracy 80%
Peramun etillseke (2002)	News headlines & keywords: the US, inflation, weak Bund, etc.	DEM/USD & JPY/USD	Boolean, TF <sup>2</sup> x IDF <sup>3</sup> & TF x CDF <sup>1</sup>	Hourly data for 22 Sept 12:00 to 27 Sept 9:00.	117	Weighting method TF x CDF performs the best with a prediction accuracy of 53% for DEM/USD
El Shazly (1997)	Euro rate on the foreign currency (USD) deposit, the spot rate of exchange, & the premium on the foreign currency	GBP/USD, DEM/USD, JPY/USD	MLP	Weekly data for the period 8 January 1988-8 April 1994	217	Best prediction accuracy 62.5% for GBP/USD & DEM/USD
Haeri (2015)	Past values of the exchange rate & indicators: EMA <sup>2</sup> , KD <sup>3</sup> , MACD <sup>4</sup> , RSI <sup>5</sup> , WMS %R <sup>6</sup>	EUR/JPY	Hidden Markov models & CART	Daily data for 2002 to June 2008	1560	71.72%
Plakanda ras (2015)	Past values of the exchange rates & market sentiment	USD/EUR, USD/JPY, USD/GBP & AUD/USD	Posts Majority (PM), SVM, Naive Bayes, KNN <sup>7</sup> , Adaboost, Logitboost, Logistic Regression, ANN	Daily data for January 2, 2013 to December 26, 2013	246	Adaboost & Naive Bayes performs the best with a prediction accuracy of 69.388% for USD/AUD & USD/GBP

<sup>1</sup> Australian Dollar<sup>2</sup> Term frequency<sup>3</sup> Inverse document frequency

Authors (Year)	Important variables	Predicted variables	Method	Data	Sample size	Result
Galeshchuk (2017)	Past values & moving averages of data	EUR/USD, GBP/USD & USD/JPY	Shallow neural network, SVM, CNN	Daily closing rates for 2010-2015	1565	CNN performs the best with a prediction accuracy of 89.55% for USD/JPY
Özorhan (2017)	Past values & technical indicators: MACD, RSI, SMA <sup>8</sup> , WMA <sup>9</sup> , CCI <sup>10</sup> , SK% <sup>11</sup>	EUR/CHF, EUR/GBP, EUR/USD, GBP/CHF, GBP/USD & USD/CHF	GA & SVM	Daily data for 2010-2015	1265	Best prediction accuracy 64% for USD/CHF
Bakhach (2016)	Past values of the exchange rates	EUR/CHF, GBP/CHF & USD/JPY	Directional Changes (DC) framework	Daily data for 1/1/2013 to 30/6/2015	607	Best prediction accuracy 84.6% for USD/JPY
Hong (2003)	Past values of the exchange rates	CAD <sup>12</sup> /USD, DEM/USD, GBP/USD, JPY/USD & FRF <sup>13</sup> /USD	Nonlinear time series models	weekly series from January 1, 1975, to December 31, 1998	1253	There exists significant nonlinearity in the mean for exchange rate changes

Source: The research findings

Although many direction forecasting models have been proposed, none of them is based on fractals, which is considered a research gap, and this study provides further analysis of this concept. This research's innovation is in

<sup>1</sup> Category discrimination frequency

<sup>2</sup> Exponential moving average

<sup>3</sup> Stochastic oscillator

<sup>4</sup> Moving average convergence divergence

<sup>5</sup> Relative strength index

<sup>6</sup> Williams %R

<sup>7</sup> K-nearest neighbors

<sup>8</sup> Simple moving average

<sup>9</sup> Weighted moving average

<sup>10</sup> Commodity Channel Index

<sup>11</sup> Stochastic K%

<sup>12</sup> Canadian Dollar

<sup>13</sup> French Franc

presenting an original model based on the Sierpinski triangle, which performs well in the forecasting of EUR/USD direction changes.

### 3 Methodology and Data

#### 3.1 Methodology

Since the Sierpinski triangle is self-similar, every new triangle ( $T_{basic}$ ) that is created is similar to the initial triangle ( $T_{initial}$ ). If each step is denoted by  $n$  ( $T_{basic}$  is created by drawing the solid lines in Figure 1 in each step), the number of triangles created in each step equals  $3^n$  (Hattori & Hattori, 1991) (Figure 1).

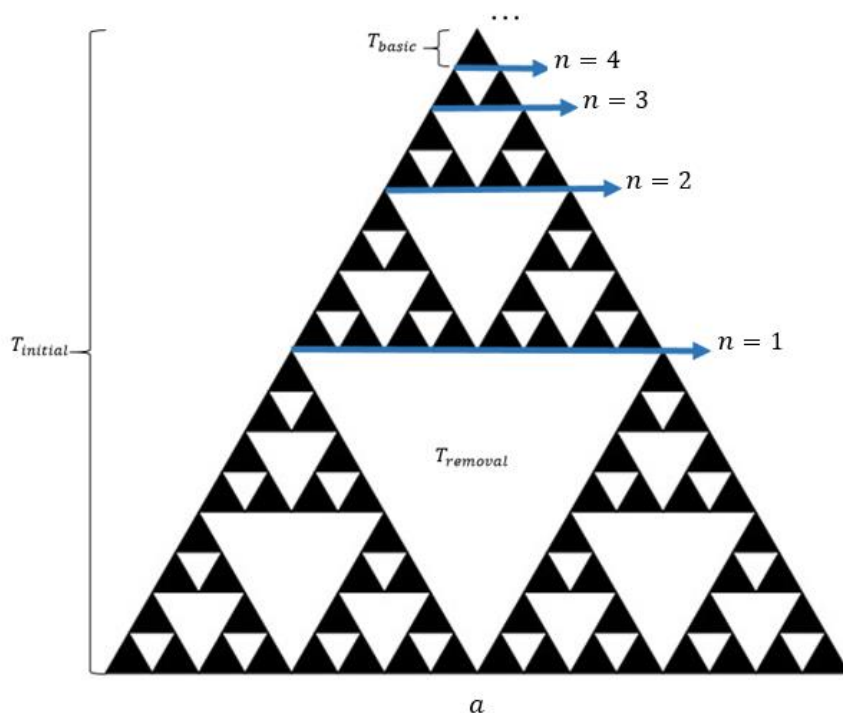


Figure 1. Sierpinski Triangle.  
Source: The research findings

The proposed model is described below.



First, Equation 1 must be proved to properly explain the concept of forecasting based on a triangle.

$$\sum_{n=1}^{\infty} S_{T_{basic}} - \sum_{n=1}^{\infty} S_{T_{removal}} = S_{T_{initial}} \quad (1)$$

Equations 2 to 5 present the process of obtaining the formula for the total area limit formula  $T_{removal}$ .

$$\text{if } n = 1 \quad S_{T_{removal}} = \frac{a^2 \sqrt{3}}{4} \quad (2)$$

$$\text{if } n = 2 \quad S_{T_{removal}} = \frac{a^2 \sqrt{3}}{16} \quad (3)$$

$$\text{if } n = 3 \quad S_{T_{removal}} = \frac{a^2 \sqrt{3}}{64} \quad (4)$$

The sequence of Equations 2, 3, 4 forms a geometric progression with  $q = \frac{1}{4}$  and  $\frac{a^2 \sqrt{3}}{4}$  as the first term towards infinity.

$$\sum_{n=1}^{\infty} S_{T_{removal}} = \frac{a^2 \sqrt{3}}{1-q} = \frac{\frac{a^2 \sqrt{3}}{4}}{1-\frac{1}{4}} = \frac{a^2 \sqrt{3}}{3} \quad (5)$$

Equations 6 to 9 show the process of achieving the total area limit formula  $T_{basic}$  (for  $n = 0, T_{basic} = T_{initial}$ )

$$\text{if } n = 0 \quad S_{T_{basic}} = a^2 \frac{\sqrt{3}}{4} \quad (6)$$

$$\text{if } n = 1 \quad S_{T_{basic}} = \frac{a^2 \sqrt{3}}{4} \quad (7)$$

$$\text{if } n = 2 \quad S_{T_{basic}} = \frac{a^2 \sqrt{3}}{16} \quad (8)$$

The sequence of Equations 6, 7, and 8 forms an infinite geometric progression with  $q = \frac{1}{4}$  and  $a^2 \frac{\sqrt{3}}{4}$  as the first term.

$$\sum_{n=1}^{\infty} S_{T_{basic}} = \frac{a^2 \sqrt{3}}{1-q} = \frac{\frac{a^2 \sqrt{3}}{4}}{1-\frac{1}{4}} = \frac{4a^2 \sqrt{3}}{3} \quad (9)$$

The difference between Equation 5 and Equation 9 equals Equation 10, which proves Equation 1.

$$\frac{4a^2}{3} \frac{\sqrt{3}}{4} - \frac{a^2}{3} \frac{\sqrt{3}}{4} = a^2 \frac{\sqrt{3}}{4} \quad (10)$$

This equation indicates that the total area of  $T_{basic}$  triangles approaches the area of the triangle  $T_{initial}$ . Triangle  $T_{initial}$  is formed from  $T_{basic}$  triangles. Therefore, Equation 11, which represents the beginning of the forecast model, can be used. As for the forecasting, triangle  $T_{initial}$  is a population with an area of  $a^2 \frac{\sqrt{3}}{4}$  and triangle  $T_{basic}$  in stage,  $n$  is the data that is forecasted with an area of  $\frac{a^2}{2^{2n}} \frac{\sqrt{3}}{4}$ . The ratio of these two values equals  $\frac{1}{2^{2n}}$ .

$$\frac{S_{T_{basic}}}{S_{T_{initial}}} = \frac{1}{2^{2n}} \quad (11)$$

Since the sum of  $T_{basic}$  areas approaches the area of  $T_{initial}$  in the infinity and forecasting can occur in each of the  $n$  stages, it is necessary to multiply the calculation by a value to minimize error with fewer steps.

$$\text{Number of } T_{basic} \text{ for } n = 3^n \quad (12)$$

$$S_{T_{basic}} \text{ in level } n = \frac{a^2}{2^{2n}} \frac{\sqrt{3}}{4} \quad (13)$$

$$a^2 \frac{\sqrt{3}}{4} = (3^n * \frac{a^2}{2^{2n}} \frac{\sqrt{3}}{4}) * \varepsilon \quad (14)$$

$$\varepsilon = \frac{2^{2n}}{3^n} \quad (15)$$

First, the most optimum  $n$  for the population ( $P$ ) must be calculated (equation 16) to carry out the forecasting.

$$n = [\ln(P)] + 1 \quad (16)$$

The  $n$  resulting from Equation 16 is in a state analyzed via Equations 17 to 20, and the results are presented in Table 2. Equation 17 indicates the ratio of  $T_{removal}$  triangles to  $T_{initial}$  in each step, and Equation 18 shows the ratio of triangles  $T_{basic}$  to  $T_{initial}$ . These two equations are used to obtain Equations 19 and 20. Equation 19 is obtained by dividing  $R_{basic}$  by  $R_{removal}$  and the exponential growth of the ratio of  $T_{basic}$  triangles to  $T_{removal}$  in each step, while Equation 20 shows the percent of the difference between the magnitudes of triangles  $T_{basic}$  to  $T_{removal}$  in each stage.

$$R_{removal} = \frac{\sum_{n=1}^n S_{T_{removal}}}{S_{T_{initial}}} = \frac{\frac{a^2 \sqrt{3}}{4}}{\frac{a^2 \sqrt{3}}{4}} = \frac{1}{2^{2n}} \quad (17)$$

$$R_{basic} = \frac{\sum_{n=1}^n S_{T_{basic}}}{S_{T_{initial}}} = \frac{3^n \times \frac{a^2 \sqrt{3}}{4}}{\frac{a^2 \sqrt{3}}{4}} = \frac{3^n}{2^{2n}} \quad (18)$$

$$\frac{R_{basic}}{R_{removal}} = 3^n \quad (19)$$

$$D = \left( \frac{R_{basic} - R_{removal}}{R_{basic}} \right) * 100 \quad (20)$$

Table 2 presents the values obtained from each of the four equations in each step. As shown, by going through the steps, the ratio of triangles  $T_{basic}$  to  $T_{removal}$  increases, reflecting the insignificance of triangles  $T_{removal}$  in the higher steps. When it reaches  $n = 7$ , the percent of the magnitude difference is approximately 100, which indicates the area of  $T_{basic}$  triangles covers the entire space. Hence, from  $n = 7$  onward, the sampling has to be carried out to take into account the effects of  $T_{removal}$  triangles.

Table 2  
Calculating the percent of magnitude difference

n	R <sub>removal</sub>	R <sub>basic</sub>	$\frac{R_{basic}}{R_{removal}}$	D (%)
1	0.25	0.75	3	66.7
2	0.5	4.5	9	88.9
3	0.75	20.25	27	96.3
4	1	81	81	98.8
5	1.25	303.75	243	99.6
6	1.5	1093.5	729	99.9
7	1.75	3827.25	2187	100
8	2	13122	6561	100
9	2.25	44286.75	19683	100
10	2.5	147622.5	59049	100
		...		
30	7.5	1.54418E+15	3 <sup>30</sup>	100
31	7.75	4.78697E+15	3 <sup>31</sup>	100
		...		

Source: The research findings

### 3.2 Data Sampling Method

Since the ratio of  $T_{removal}$  triangles to  $T_{initial}$  triangles is  $\frac{1}{3}$  in each stage, sampling is performed with the following formula, and the steps are continued until number  $n$  is below 7 considering  $X_{i+1}$  and the calculated  $n$  value.

$$X_i: P \quad (21)$$

$$X_{i+1} = X_i - \frac{X_i}{3} \quad (22)$$

Based on all the equations, the final forecasting model is described below (equation 23).

$P$ : Population (total data)

$\hat{E}_{j+1}$ : The predicted value at time  $j + 1$

$E_j$ : The amount of data at time  $j$

$\varepsilon$ : Error

$$n = [\ln(P)] + 1$$

$$\hat{E}_{j+1} = \frac{1}{2^{2n}} \times (\sum_1^j E_j) \times P \times \frac{1}{\varepsilon} \quad (23)$$

The flowchart of the proposed model is also presented in Figure 2.

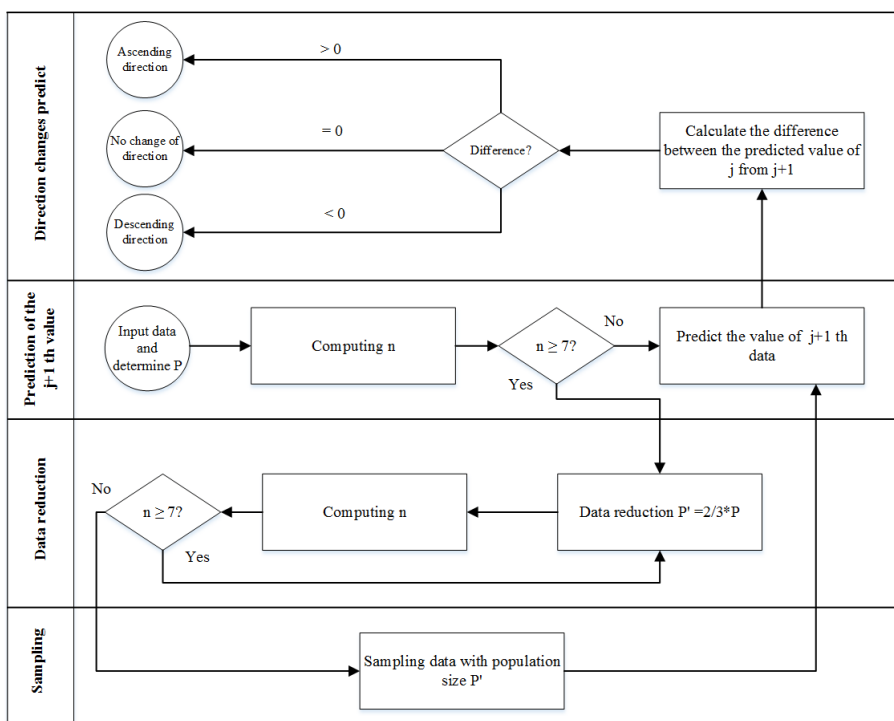


Figure 2. Flowchart of the Proposed Model.

Source: The research findings

### 3.3 Data Description

In this study, the EUR/USD closed price data from the period before 28/02/2020 is used. The movement direction of the last 300 data (one year from 03/01/2019 to 28/02/2020) in three categories, namely training (70% data equal to 210 data), testing (20% data equal to 60 data), and validation (10% data equal to 30 data) categories is forecasted based on previous data. The stability and reliability (accuracy) of the model can be checked by using these categories. The training data used to estimate the model parameters during the learning process (population size or  $P$ -value). The testing data is independent of the training data but follows the same probability distribution that is used only to assess the model performance. The validation data (development set) are used to tune the model hyper-parameters, which is necessary, in addition to the training and testing data to avoid overfitting. Moreover, the importance of training data is that models which have poor performance on training data will never generalize testing data well. Models that perform well on training data might or might not generalize well.

The trend of the EUR/USD in the time of study is presented in Figure 3.

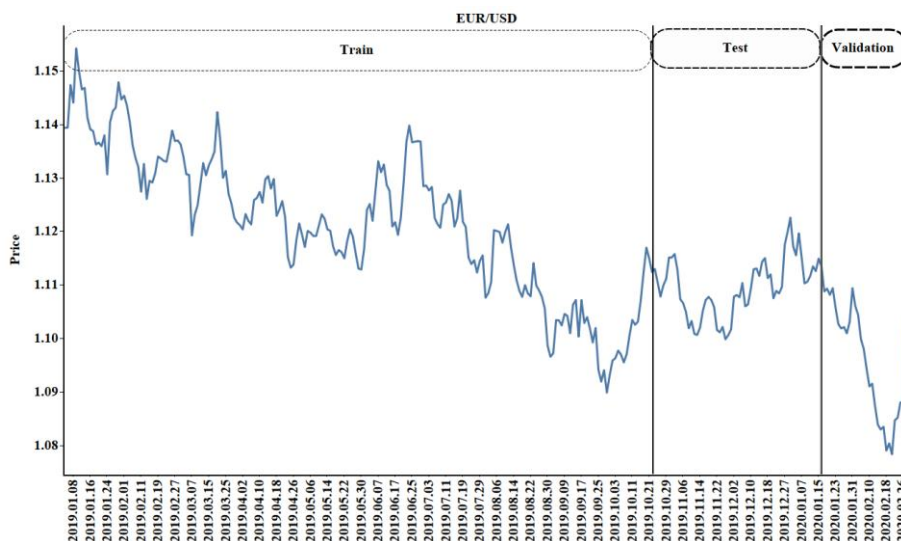


Figure 3. The Trend of EUR/USD in the Time of Study.

Source: The research findings with Tableau software from data of Meta Trader 5

To improve the assessment, the results of the proposed model are compared with the methods introduced in (Galeshchuk & Mukherjee, 2017)

using 1565 data from 01/01/2010 to 15/06/2015 with 1043 data serving as training data and 522 data serving as testing data. Data is obtained from Meta Trader 5 software.

#### **4 Result and Discussion**

To assess the results, the EUR/USD data described in part one of Section 3.3 is classified into three categories, namely training (70% of data), testing (20% of data) and validation (10% of data), and the model is implemented on the entire population. Table 3 shows the population sizes for which the direction forecasting accuracy for the training data is over 0.6. The direction forecasting accuracy (equation 27) is calculated as follows.

$\hat{E}_{j+1}$ : The predicted value at time  $j + 1$

$E_j$ : The amount of data at time  $j$

$\hat{D}_{j+1}$ : Changes in the predicted direction at time  $j + 1$

$D_{j+1}$ : Changes in the actual direction of the data at time

$J$ : Total number of predicted data

$Dif_{j+1}$ : The difference between the real and the predicted change of direction

$AC$ : Direction forecasting accuracy

$$\hat{D}_{j+1} = \hat{E}_{j+1} - E_j = \begin{cases} 0, & \hat{D}_{j+1} < 0 \\ 1, & \hat{D}_{j+1} \geq 0 \end{cases} \quad (24)$$

$$D_{j+1} = E_{j+1} - E_j = \begin{cases} 0, & D_{j+1} < 0 \\ 1, & D_{j+1} \geq 0 \end{cases} \quad (25)$$

$$Dif_{j+1} = \hat{D}_{j+1} - D_{j+1} = \begin{cases} 0, & Dif_{j+1} \neq 0 \\ 1, & Dif_{j+1} = 0 \end{cases} \quad (26)$$

$$Ac = \frac{\sum_J Dif_{j+1}}{J} \quad (27)$$

Table 3

The direction forecasting accuracy of more than 0.6 for  $P$  values in the training data

Accuracy train	Train	
	n train	P train
0.608	7	669
0.608	7	773
0.612	7	922
0.612	7	1093
0.603	8	1928
0.608	8	2530

Source: The research finding

To increase the reliability of the results listed in Table 3, the model is applied to the testing data for  $P$  values. The  $P$ s for which the direction accuracy difference between the training and testing data is below 0.05 are listed in Table 4.

Table 4

*The <0.05 direction forecasting accuracy difference between the testing and training data for P values*

<b>Difference</b>	<b>Accuracy test</b>	<b>Test</b>	
		<b>n test</b>	<b>P test</b>
0.007	0.610	8	1928
0.031	0.576	8	2530

Source: The research finding

According to Table 4,  $P=1928$  led to the best result. A similar process is applied to the validation data to increase reliability. The direction difference between the testing and validation data is 0.018, which is smaller than 0.05 and presents a reliable result for the existing values (Table 5).

Table 5

*The <0.05 forecasting accuracy difference between the testing and validation data for P=1928*

<b>Difference</b>	<b>Accuracy valid</b>	<b>validation</b>	
		<b>n test</b>	<b>P test</b>
0.018	0.621	8	1928

Source: The research finding

Since the model results for  $n=8$  are the best results, sampling has to be performed for each iteration of the model based on the flowchart depicted in Figure 2 and the research methodology section. Since this process is stochastic, the model is applied to 300 data for 30 iterations for the best  $P$ -value to ensure the results. The results are also analyzed with three approaches.

- 1) In the first approach, which is named mean one group forecasting (MOGF), the model is implemented 30 times on 210 training data, 60 testing data, 30 validation data, and the mean direction accuracy for 210, 60 30 data is calculated per iteration. Finally, the mean and the best value are calculated for 30 iterations per category (Table 6).
- 2) The training data is classified into seven 30-tuple categories, the testing data is classified into two 30-tuple categories, and validation data is divided into one 30-tuple category monthly. After that, the mean (second approach) and the best (third approach) direction forecasting accuracy are calculated for 30 iterations for each category, and the following two approaches can be adopted.



- 1) In the second approach, which is named the mean of 30-tuple categories' mean forecasting (MOCMF), the mean of the mean values of the categories' direction forecasting accuracy is calculated (The representative of each month is a mean value).
- 2) In the third approach, which is named the mean of 30-tuples categories' best forecasting (MOCBF), the mean of the best values of the direction forecasting accuracy of the categories is calculated (The representative of each month is the maximum value) (Table 7).

Table 6

*The mean and the direction forecasting accuracy for 30 runs and  $P=1928$*

Moods	Accuracy Train		Accuracy Test		Accuracy Valid	
	best	mean	best	mean	best	mean
1	0.603	0.495	0.678	0.513	0.655	0.522

Source: The research finding.

Table 7

*The mean and best direction forecasting accuracy in the second and third approaches for 30 runs and  $p=1928$*

Moods	Accuracy Train			Accuracy Test			Accuracy Valid	
	Mean best	of best	mean	Mean best	of best	mean	best	mean
2 & 3	0.705	0.733	0.504	0.667	0.667	0.515	0.6552	0.5218

Source: The research finding.

These results show the above 50% and 60% accuracy of the model in the mean state and the best state of EUR/USD direction changes forecasting within one year. The similar results of three categories named training, testing, and validation show the model's stability and reliability.

Table 8 presents the comparison of the results of the proposed model using 3 approaches with the results of (Galeshchuk & Mukherjee, 2017) in forecasting the EUR/USD direction changes. The importance of the proposed model and the results lies in the fact that concerning the mean direction accuracy, it outperforms the MC<sup>1</sup>, ARIMA<sup>2</sup>, ETS<sup>3</sup>, ANN, ANN (MA)<sup>4</sup>, and SVM models in the third approach, while as regards the best value it

<sup>1</sup> Majority class

<sup>2</sup> Autoregressive integrated moving average

<sup>3</sup> Exponential smoothing

<sup>4</sup> ANN model trained on moving averages as input features

outperforms the MC, ARIMA, ETS, ANN, ANN (MA), and SVM modes in the second and third approaches. Although these models did not improve the results compared to the CNN model, the proposed method's performance is not influenced by the training input features. From a trading perspective, this approach is easier to implement than approaches such as CNN, raises the trading win-rates and provides better returns, leading to risk management and profitability.

Table 8

*Comparing the results of the proposed method (for  $P=1094$ ) and the other methods*

Methods	Accuracy test (%)		
	Mean	Best	Mean of best
MC	40.30	76.80	-
ARIMA	60.05	64.38	-
ETS	52.05	58.93	-
ANN	58.33	71.83	-
ANN (MA)	65.40	76.83	-
SVM	65.13	76.06	-
CNN	75.28	89.55	-
Proposed method (MOGF)	1 50.6	56.7	-
Proposed method (MOCF)	2 50.55	79.31	-
Proposed method (BOCF)	3 -	79.31	69.15

Source: The research finding and (Galeshchuk & Mukherjee, 2017)

Table 9 provides full details of the proposed model and compares it with previous work (Table 1) from different perspectives.

Table 9

*Full details of the proposed model*

	Important variables	Predicted variables	Method	Data	Sample size	Result
The proposed method	Past values of the exchange rates	EUR/USD exchange rates	Sierpinski Method	Daily data 03/01/2019 28/02/2020	for to 300	Prediction accuracy 79.31 %

Source: The research finding

## 5 Conclusions

The price and direction forecasting models in the financial and stock markets lead to a decrease in risks and proper risk management. The analysis of the fractals' features and structure has also led to the development of forecast models for the same purpose. The Sierpinski triangle is a fractal, and an innovative model is provided to forecast direction changes of currency pairs, especially EUR/USD, with an approach different from other studies following a review of the literature on the Sierpinski triangle in this study. This model's performance in forecasting any value is determined by the number of data used in forecasting ( $P$ ). Hence, the data is first classified into three categories: the training, testing, and validation categories, and the model is implemented for different  $P$  values. The best  $P$ -value is selected such that the model has a direction forecasting accuracy of above 0.6 in the training data and a difference smaller than 0.05 in the testing data. Afterward, the model results are analyzed for the best  $P$ -value using 300 EUR/USD closed price data in three categories: the training, testing, and validation categories. The mean direction forecasting accuracy and the best state are above 50 and 60%, respectively. These results reflect the performance and reliability of the model results within one year. Besides, the results of the proposed method are compared with the other methods considering these three approaches to improve the assessment results: 1- mean one group forecasting; 2- forecasting using 30-tuple categories considering the mean of categories; 3- forecasting using 30-tuple categories considering the mean of the best state of the categories. The third approach yields better results than the two approaches above and the econometric, time series, and other machine learning methods.

The proposed method's importance also lies in the fact that the training input features do not influence it.

For future research, this model can be developed such that it stages a good performance in the forecasting of prices, and it can function based on multivariate regression coefficients.

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