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Application of Malmquist Index in Two-Stage DEA for Measurement of Productivity Growth

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The purpose of this paper is to develop an output oriented methodology for calculating productivity growth by using Malmquist productivity index (MPI) and two different data envelopment analysis (DEA) views (optimistic and pessimistic) simultaneously, and apply it to five Iranian Commercial Banks over the four time period (2009-2013). Consequently, we have proposed a new approach called the double frontiers two-stage DEA or DFTDEA for simultaneous measurement of the MPI from both different DEA views. Furthermore, this paper has used two-stage DEA with reference to the variable return to scale technology (VRS) and applied a new viewpoint to measure the overall efficiency of the process.

Keywords: Two-stage DEA, Malmquist productivity index, Iranian Commercial Banks
JEL Classification*:* E01, E17, E37

1 Introduction

DEA is a linear programming and non-parametric based methodology to measure the relative efficiency which can measure homogeneous multiple inputs and outputs and can also evaluate decision-making units (DMU) both qualitatively and quantitatively. DEA was proposed by Charnes, Cooper and Rhodes (1978) in order to apply linear programming to estimate an empirical production technology frontier for the first time which later was known as the CCR model from their acronyms. The evolutionary form of CCR model was suggested by Banker, Charnes and Cooper (1984) which later was known as the BCC model from their acronyms. Since then, there have been several books and papers written on DEA models or their application was developed by a large number of researchers. Orientation (Input/Output), returns to scale

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(Constant return to scale; CRS/Variable returns to scale; VRS), disposability and different aspects which can be seen in these models.

Given that many production processes and services in real issues are interdependent and have several complexities, moreover traditional DEA models consider the DMUs as black boxes without internal communication processes, it is necessary that we adopt models compatible with these situations for a more detailed evaluation of the DMUs under discussion. One issue debated widespread in recent years to solve this matter is that DMU with two-stage network structure. Consider the fundamental two stage process which is shown schematically in Figure 1, and whereas there exist *n* DMUs to be evaluated for DMU_j (j=1,..., n) which has two subprocesses with m inputs x_{ij} (i=1,...,m) in the first stage, D intermediate measures z_{dj} (d=1,...,D) as outputs of the first stage and inputs of the second stage, s outputs y_{rj} (r=1,...,s) in the second stage (Figure 1).

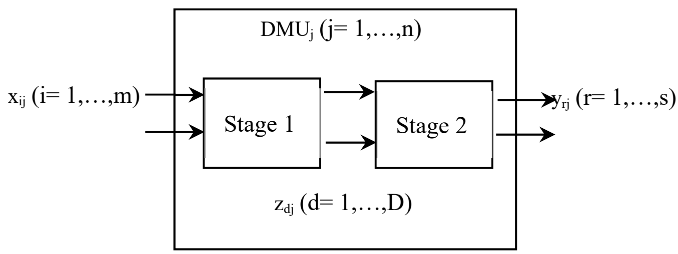

Figure 1. Fundamental Two-stage Process

According to the above-mentioned matters, the efficiencies of the first and the second processes can be calculated as a single process by using the conventional DEA methodology. Ray (1991) and Fried, Lovell and Eeckaut (1993) have raised the two-stage DEA models so that using the standard model with desirable factors in the first stage and analysis regression in the second stage, however, undesirable factors were considered in these models as independent variables. Kao (1995) showed that overall efficiency under linear production frontiers is a weighted arithmetic mean of the efficiencies of the outputs. Similarly, he decomposed the overall efficiency with respect to input factors as well, and some results were derived. Seiford and Zhu (1999) examined the performance of 55 U.S. commercial banks via a two-stage process that separates profitability and marketability as results of the first and second stage, respectively. Fare and Grosskopf (2000) proposed a method for decomposing the black boxes of the traditional DEA to evaluate organizational performance and its components. The proposed general structure of network DEA model can be applied to various situations. Sexton and Lewis (2003) used standard two-stage DEA models for evaluating process performance on the major league baseball. Kao and Hwang (2008) modified the conventional DEA model by proposing a relational two-stage DEA model and tested it on 24 non-life insurance companies. Chen, Cook, Li and Zhu (2009) developed an additive efficiency decomposition approach wherein the overall efficiency is expressed as the weighted sum of the efficiencies of the individual stages. Furthermore, the two-stage DEA models have also been applied to a large number of researchers to measure the performance of information technology, supply chain, R&D, bank industry, aviation industry, etc. It must be noted that all of the above researches are not applicable for measuring efficiency over time periods.

In recent years, measuring productivity changes over time has been a very important issue among the researchers who have analyzed the performance of units. Malmquist productivity index (MPI) was originally defined by Professor Sten Malmquist (1953) as a quality index for analyzing the consumption of production resources. MPI is based on the concept of the production function and makes use of distance functions to measure productivity changes and also it can be defined by using input and output oriented distance functions. Hence, MPI as a concept is compatible and coincident with the DEA methodology. MPI approach was proposed and entered for the first time in productivity literature by Caves, Christensen and Diewert (1982). Hereupon, Fare, Grosskopf and Russell (1998) used DEA techniques to compute MPI. Afterwards, Fare, Grosskopf, Norris and Zhang (1994) developed an output oriented non-parametric based methodology for calculating productivity changes and applied it to industrialized countries and was later named it FGNZ decomposition from their acronyms. In this connection, they combined ideas from the efficiency measurement by Farrell (1957) and the productivity measurement by Caves et al. (1982) and finally constructed the DEA-based MPI to decompose it into efficiency changes and technology changes (frontier shifts) over time.

In this regard, we developed FGNZ decomposition to output oriented for measuring productivity growth analyzing in five Iranian commercial banks. Grifell-Tatje and Lovell (1997) used the traditional BCC model to measure the productivity growth of Spanish banking system with a single process. Ray and Desli (1997) took a comment on the FGNZ approach and applied it to the same countries which had been observed in adjacent years. Chen and Yeh (2000) extended the output oriented DEA-based MPI with reference technology exhibiting VRS frontier and applied it to 34 commercial banks in Taiwan. Balk (2001) developed a generic measure of scale efficiency for multiple inputs and multiple output firms, and also combined measures of technological change, technical efficiency change and scale efficiency change into a primal measure of productivity change. Mukherjee, Ray and Miller (2001) isolated the contributions of technical change, technical efficiency change and scale efficiency change to productivity growth using DEA method with VRS technology for measuring the MPI's input distance functions and applied them to 201 large US commercial banks over the initial post-deregulation period during 1984-1990. Wang and Lan (2011) proposed a new approach based on double frontiers input-oriented DEA (DFDEA) based MPI. The MPI measured from DFDEA had geometrically been averaged to generate an integrated MPI. Furthermore, the DEA based MPI models with VRS reference technology has also been applied by a number of researchers such as Portela and Thanassoulis (2006).

In this paper, we first propose an overall efficiency for two stage output-oriented DEA models with VRS reference technology which can be used over time. The rest of the paper is organized as follows: Section 2 proposes the relational two-stage DEA based MPI model. Section 3 defines the optimistic and pessimistic two-stage DEA based MPI and measurement models (OMPI and PMPI), respectively. The empirical illustration is presented and discussed in section 4. In Section 5, the proposed models and MPIs are applied to the productivity analysis of the five Iranian commercial banks. Paper conclusions are presented in the last section.

2 Relational Two-stage DEA Model-based MPI

Consider n DMUs ($DMU_{j:\ j}=1,\ldots,n$) using m inputs x_{ij} ($i=1,\ldots,m$) to generate D outputs z_{dj} ($d=1,\ldots,D$) in the first stage and D inputs z_{dj} ($d=1,\ldots,D$) as intermediate measures to generate s outputs y_{rj} ($r=1,\ldots,s$) in the second stage. Let x_{ij}, z_{dj} and y_{rj} are the i^{th} input, D^{th} intermediate measure and s^{th} output of the j^{th} DMU, respectively. The efficiency of the DMU_j through the conventional output-oriented DEA model under the assumption of variable return to scale for DMU_p is measured as follows:

$$Min\ (\sum_{i=1}^{m} v_i x_{ip} + \omega)/\sum_{r=1}^{s} u_r y_{rp} \qquad (1)$$
$$s.t.\quad \sum_{r=1}^{s} u_r y_{rp}\ /(\sum_{i=1}^{m} v_i x_{ij} + \omega) \le 1 \quad ,j = 1, \dots, n$$
$$v_i, u_r \ge \varepsilon, \qquad i = 1, \dots, m, \qquad r = 1, \dots, s$$

Regarding the above-mentioned model for measuring the efficiency of DMU_p with a single process and considering the fundamental two-stage process shown in Figure 1, we can use it for measuring the efficiencies of DMU_p in the two individual stages (first and second stages) as:

$$E_p^1 = Min\ (\sum_{i=1}^{m} v_i x_{ip} + \omega)/\sum_{d=1}^{D} w_d z_{dp} \qquad (2)$$
$$E_p^2 = Min\ (\sum_{d=1}^{D} w_d z_{dp} + \omega')/\sum_{r=1}^{s} u_r y_{rp}$$

Based on the above efficiencies about the first and second stages of DMU_j, the overall efficiency of DMU_j in the entire two-stage process has been defined by several researchers as:

- Azizi and Kazemi Matin (2010) extended the Kao and Hwang (2008) method to the VRS assumption and defined the overall efficiency of DMU_j as the product of the efficiencies of the two sub-processes that is to say$E_p = E_p^1 \times E_p^2$. It should be necessary noted that aforesaid relational efficiency (E_p) can only be applied for efficiency measurement in single period and not practical for performance evaluation in the multiple period.
- Chen, Liang and Zhu (2009) extended the Chen et al. (2009) method to the VRS assumption and proposed the overall efficiency as weighted summary of the two individual stages. It must be noted that like the above mentioned method, the aforesaid proposed method can be applied just for efficiency measurement in a single period and not practical for performance evaluation in the multiple periods.
- Considering the Figure 1, Wang and Chin (2010) defined the overall efficiency under both CRS and VRS technology where the intermediate measures z_{dj} (d=1,…,D) serve as both inputs and outputs of DMU_p at the same time (Figure 2). They introduced the $\lambda_1, \lambda_2 > 0$ as set of relative importance weighs of the two-stages such that $\lambda_1 + \lambda_2 = 1$.

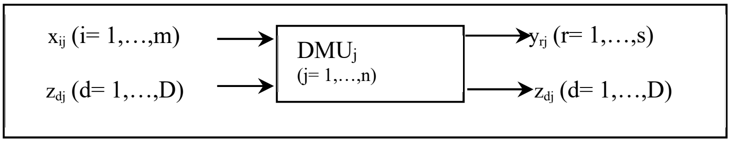

Figure 2. Wang and Chin Proposed Transformation from Fundamental Two-stage Process to a Single Process

– Considering the Figure 1, Saleh, Hosseinzadeh Lotfi, Toloie Eshlaghy and Shafiee (2011) defined the overall efficiency with CRS reference technology which can be extended to the VRS technology where the intermediate measures z_{dj} (d=1,…,D) serve as outputs of DMU_p (Figure 3).

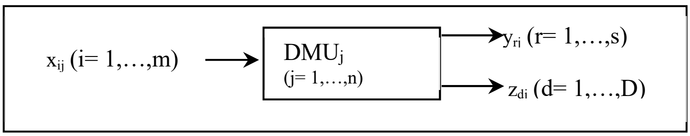

Figure 3. Saleh et al. Proposed Transformation from Fundamental Two-stage Process to a Single Process

According to the above and considering the Figure 1, we define the overall efficiency of DMU_j such that the intermediate measures z_{dj} (d=1,…,D) serve as inputs of DMU_p. Thereupon, the proposed optimistic output oriented two-stage DEA model under VRS assumption can be constructed as bellow:

$$\begin{aligned} & Min\ (\textstyle\sum_{i=1}^{m} v_i x_{ip} + \sum_{d=1}^{D} w_d z_{dp} + \omega)/\sum_{r=1}^{s} u_r y_{rp} \\ & s.t.\quad \textstyle\sum_{r=1}^{s} u_r y_{rj} /(\sum_{i=1}^{m} v_i x_{ij} + \sum_{d=1}^{D} w_d z_{dj} + \omega) \le 1 \quad ,j = 1,\dots,n \\ & \qquad u_r, v_i, w_d \ge \varepsilon, i = 1,\dots,m, r = 1,\dots,s, d = 1,\dots,D \end{aligned} \tag{4}$$

Now, the dual model (4) is:

$$\begin{aligned} & Max\ \varphi_p \\ & \text{s.t. } \textstyle\sum_{j=1}^{n} \lambda_j\, x_{ij} \le x_{ip} && ,i = 1,\dots,m \\ & \textstyle\sum_{j=1}^{n} \lambda_j\, z_{dj} \le z_{dp} && ,d = 1,\dots,D \\ & \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \ge \varphi_p y_{rp} && ,r = 1,\dots,s \\ & \textstyle\sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \ge 0,\ \varphi_p \text{ free, j=1,…, n} \end{aligned} \tag{5}$$

Accordingly, the proposed pessimistic model can be constructed as follows:

$$Max\ (\sum_{i=1}^{m} v_i x_{ip} + \sum_{d=1}^{D} w_d z_{dp} + \omega) / \sum_{r=1}^{s} u_r y_{rp} \tag{6}$$
$$s.t.\quad \sum_{r=1}^{s} u_r y_{rj} / (\sum_{i=1}^{m} v_i x_{ij} + \sum_{d=1}^{D} w_d z_{dj} + \omega) \geq 1 \quad ,j = 1, \dots, n$$
$$u_r, v_i, w_d \geq \varepsilon, i = 1, \dots, m, r = 1, \dots, s, d = 1, \dots, D$$

Now, the dual model (6) is:

$$Min\ \varphi_p \tag{7}$$
$$s.t.\quad \sum_{j=1}^{n} \lambda_j x_{ij} \geq x_{ip} \quad ,i = 1, \dots, m$$
$$\sum_{j=1}^{n} \lambda_j z_{dj} \geq z_{dp} \quad ,d = 1, \dots, D$$
$$\sum_{j=1}^{n} \lambda_j y_{rj} \leq \varphi_p y_{rp} \quad ,r = 1, \dots, s$$
$$\sum_{j=1}^{n} \lambda_j = 1$$
$$\lambda_j \geq 0, \varphi_p \text{ free, j=1,…,n}$$

In addition to the above relational efficiency in two different views (optimistic and pessimistic), we can apply it for evaluating the performance changes for DMUs between two periods for multi-period problems. For this purpose, we use the Malmquist productivity index (MPI), because that is an index which has been broadly used by researchers for measuring the performance changes and it can be combined with the DEA models.

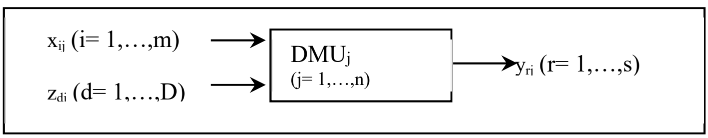

Figure 4. Transformation from Fundamental Two-stage Process to a Single Process

3 The Optimistic and Pessimistic Two-stage DEA-based MPI

According to the considerations stipulated in the above section, concerning inputs, intermediate measures and outputs of the two-stage processes, denote x_{ij}^t, z_{dj}^t and y_{rj}^t as the process data at the time period *t* and x_{ij}^{t+1}, z_{dj}^{t+1} and y_{rj}^{t+1} at the time period *t*+1, respectively. For measuring the optimistic two-stage DEA based MPI, we should solve the following linear programming problems for two single period and two mixed period measures:

1) The first single period measures the efficiencies of DMU_p in time period t

$$D_1^t(x_p^t \text{‘} z_p^t) = Max\ \alpha_p^t(t) \tag{8}$$
s.t. $\sum_{j=1}^{n} \mu_j^t\, x_{ij}^t \leq x_{ip}^t \qquad , i = 1, \dots, m$
$\sum_{j=1}^{n} \mu_j^t\, z_{dj}^t \geq \alpha_p^t(t) z_{dp}^t \qquad , d = 1, \dots, D$
$\sum_{j=1}^{n} \mu_j^t = 1$
$\mu_j^t \geq 0$, $\alpha_p^t(t)$ free, j=1,…, n

$$D_2^t(z_p^t \text{‘} y_p^t) = Max\ \beta_p^t(t) \tag{9}$$
s.t. $\sum_{j=1}^{n} \delta_j^t\, z_{dj}^t \leq z_{dp}^t \qquad , d = 1, \dots, D$
$\sum_{j=1}^{n} \delta_j^t y_{rj}^t \geq \beta_p^t(t) y_{rp}^t \qquad , r = 1, \dots, s$
$\sum_{j=1}^{n} \delta_j^t = 1$
$\delta_j^t \geq 0$, $\beta_p^t(t)$ free, j=1,…, n

$$D_O^t(x_p^t \text{‘} y_p^t) = Max\ \varphi_p^t(t) \tag{10}$$
s.t. $\sum_{j=1}^{n} \lambda_j^t\, x_{ij}^t \leq x_{ip}^t \qquad , i = 1, \dots, m$
$\sum_{j=1}^{n} \lambda_j^t\, z_{dj}^t \leq z_{dp}^t \qquad , d = 1, \dots, D$
$\sum_{j=1}^{n} \lambda_j^t\, y_{rj}^t \geq \varphi_p^t(t) y_{rp}^t \qquad , r = 1, \dots, s$
$\sum_{j=1}^{n} \lambda_j^t = 1$
$\lambda_j^t \geq 0$, $\varphi_p^t(t)$ free, j=1,…, n

2) The second single period measures the efficiencies of DMU_p in time period t+1

$$D_1^{t+1}(x_p^{t+1} \text{‘} z_p^{t+1}) = Max\ \alpha_p^{t+1}(t+1) \tag{11}$$
s.t. $\sum_{j=1}^{n} \mu_j^{t+1}\, x_{ij}^{t+1} \leq x_{ip}^{t+1} \qquad , j = 1, \dots, n$
$\sum_{j=1}^{n} \mu_j^{t+1}\, z_{dj}^{t+1} \geq \alpha_p^{t+1}(t+1) z_{dp}^{t+1} \qquad , d = 1, \dots, D$
$\sum_{j=1}^{n} \mu_j^{t+1} = 1$
$\mu_j^{t+1} \geq 0$, $\alpha_p^{t+1}(t+1)$ free, j=1,…, n

$$D_2^{t+1}(z_p^{t+1}\text{‘}y_p^{t+1}) = Max\ \beta_p^{t+1}(t+1) \tag{12}$$

s.t. $\sum_{j=1}^{n} \delta_j^{t+1} z_{dj}^{t+1} \leq z_{dp}^{t+1}$ $\quad ,d = 1, \ldots, D$

$\sum_{j=1}^{n} \delta_j^{t+1} y_{rj}^{t+1} \geq \beta_p^{t+1}(t+1) y_{rp}^{t+1}$ $\quad ,r = 1, \ldots, s$

$\sum_{j=1}^{n} \delta_j^{t+1} = 1$

$\delta_j^{t+1} \geq 0$, $\beta_p^{t+1}(t+1)$ free, j=1,…, n

$$D_O^{t+1}(x_p^{t+1}\text{‘}y_p^{t+1}) = Max\ \varphi_p^{t+1}(t+1) \tag{13}$$

s.t. $\sum_{j=1}^{n} \lambda_j^{t+1} x_{ij}^{t+1} \leq x_{ip}^{t+1}$ $\quad ,i = 1, \ldots, m$

$\sum_{j=1}^{n} \lambda_j^{t+1} z_{dj}^{t+1} \leq z_{dp}^{t+1}$ $\quad ,d = 1, \ldots, D$

$\sum_{j=1}^{n} \lambda_j^{t+1} y_{rj}^{t+1} \geq \varphi_p^{t+1}(t+1) y_{rp}^{t+1}$ $\quad ,r = 1, \ldots, s$

$\sum_{j=1}^{n} \lambda_j^{t+1} = 1$

$\lambda_j^{t+1} \geq 0$, $\varphi_p^{t+1}(t+1)$ free, j=1,…, n

3) The first mixed period measures the efficiencies of DMU_p in time period *t* by using the frontier of the time period *t*+1 instead of *t*

$$D_1^{t+1}(x_p^{t}\text{‘}z_p^{t}) = Max\ \alpha_p^{t+1}(t) \tag{14}$$

s.t. $\sum_{j=1}^{n} \mu_j^{t+1} x_{ij}^{t+1} \leq x_{ip}^{t}$ $\quad ,i = 1, \ldots, m$

$\sum_{j=1}^{n} \mu_j^{t+1} z_{dj}^{t+1} \geq \alpha_p^{t+1}(t) z_{dp}^{t}$ $\quad ,d = 1, \ldots, D$

$\sum_{j=1}^{n} \mu_j^{t+1} = 1$

$\mu_j^{t+1} \geq 0$, $\alpha_p^{t+1}(t)$ free, j=1,…, n

$$D_2^{t+1}(z_p^{t}\text{‘}y_p^{t}) = Max\ \beta_p^{t+1}(t) \tag{15}$$

s.t. $\sum_{j=1}^{n} \delta_j^{t+1} z_{dj}^{t+1} \leq z_{dp}^{t}$ $\quad ,d = 1, \ldots, D$

$\sum_{j=1}^{n} \delta_j^{t+1} y_{rj}^{t+1} \geq \beta_p^{t+1}(t) y_{rp}^{t}$ $\quad ,r = 1, \ldots, s$

$\sum_{j=1}^{n} \delta_j^{t+1} = 1$

$\delta_j^{t+1} \geq 0$, $\beta_p^{t+1}(t)$ free, j=1,…, n

$$D_O^{t+1}(x_p^t ‘y_p^t) = Max\ \varphi_p^{t+1}(t) \quad (16)$$
s.t. $\sum_{j=1}^{n} \lambda_j^{t+1} x_{ij}^{t+1} \le x_{ip}^t \qquad ,i = 1, \dots, m$
$\sum_{j=1}^{n} \lambda_j^{t+1} z_{dj}^{t+1} \le z_{dp}^t \qquad ,d = 1, \dots, D$
$\sum_{j=1}^{n} \lambda_j^{t+1} y_{rj}^{t+1} \ge \varphi_p^{t+1}(t) y_{rp}^t \qquad ,r = 1, \dots, s$
$\sum_{j=1}^{n} \lambda_j^{t+1} = 1$
$\lambda_j^{t+1} \ge 0, \varphi_p^{t+1}(t)$ free, j=1,…, n

4) The second mixed period measures the efficiencies of DMU_p in time period t+1 while using the frontier of the time period t instead of t+1

$$D_1^t(x_p^{t+1} ‘z_p^{t+1}) = Max\ \alpha_p^t(t+1) \quad (17)$$
s.t. $\sum_{j=1}^{n} \mu_j^t x_{ij}^t \le x_{ip}^{t+1} \qquad ,i = 1, \dots, m$
$\sum_{j=1}^{n} \mu_j^t z_{dj}^t \ge \alpha_p^t(t+1) z_{dp}^{t+1} \qquad ,d = 1, \dots, D$
$\sum_{j=1}^{n} \mu_j^t = 1$
$\mu_j^t \ge 0, \alpha_p^t(t+1)$ free, j=1,…, n

$$D_2^t(z_p^{t+1} ‘y_p^{t+1}) = Max\ \beta_p^t(t+1) \quad (18)$$
s.t. $\sum_{j=1}^{n} \delta_j^t z_{dj}^t \le z_{dp}^{t+1} \qquad ,d = 1, \dots, D$
$\sum_{j=1}^{n} \delta_j^t y_{rj}^t \ge \beta_p^t(t+1) y_{rp}^{t+1} \qquad ,r = 1, \dots, s$
$\sum_{j=1}^{n} \delta_j^t = 1$
$\delta_j^t \ge 0, \beta_p^t(t+1)$ free, j=1,…, n

$$D_O^t(x_p^{t+1} ‘y_p^{t+1}) = Max\ \varphi_p^t(t+1) \quad (19)$$
s.t. $\sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le x_{ip}^{t+1} \qquad ,i = 1, \dots, m$
$\sum_{j=1}^{n} \lambda_j^t z_{dj}^t \le z_{dp}^{t+1} \qquad ,d = 1, \dots, D$
$\sum_{j=1}^{n} \lambda_j^t y_{rj}^t \ge \varphi_p^t(t+1) y_{rp}^{t+1} \qquad ,r = 1, \dots, s$
$\sum_{j=1}^{n} \lambda_j^t = 1$
$\lambda_j^t \ge 0, \varphi_p^t(t+1)$ free, j=1,…, n

Further to the above linear programming for optimistic two-stage DEA-based MPI, for measuring the pessimistic point of view, we just need to do the following changes to the above linear programming:

a) Change all targets from *Max* form to the *Min* form
b) Change all ≤ signs to the ≥ sign and vice versa

Then, we can use Far et al. (1994) and Ray and Desli (1997) proposed optimistic MPI decomposition for the first stage, second stage and whole process as follows:

$$OMPI_1 = \frac{D_1^{t+1}(x^{t+1},z^{t+1})}{D_1^{t}(x^{t},z^{t})} \left[\frac{D_1^{t}(x^{t+1},z^{t+1})}{D_1^{t+1}(x^{t+1},z^{t+1})} \frac{D_1^{t}(x^{t},z^{t})}{D_1^{t+1}(x^{t},z^{t})}\right]^{1/2} \quad (20)$$

$$OMPI_2 = \frac{D_2^{t+1}(z^{t+1},y^{t+1})}{D_2^{t}(z^{t},y^{t})} \left[\frac{D_2^{t}(z^{t+1},y^{t+1})}{D_2^{t+1}(z^{t+1},y^{t+1})} \frac{D_2^{t}(z^{t},y^{t})}{D_2^{t+1}(z^{t},y^{t})}\right]^{1/2} \quad (21)$$

$$OMPI_O = \frac{D_O^{t+1}(x^{t+1},y^{t+1})}{D_O^{t}(x^{t},y^{t})} \left[\frac{D_O^{t}(x^{t+1},y^{t+1})}{D_O^{t+1}(x^{t+1},y^{t+1})} \frac{D_O^{t}(x^{t},y^{t})}{D_O^{t+1}(x^{t},y^{t})}\right]^{1/2} \quad (22)$$

According to the above-mentioned equations (20, 21 & 22), the first factor on the right-hand side (efficiency change) can be decomposed to the optimistic pure efficiency change (OPEC) and optimistic scale efficiency change (OSEC), furthermore, the optimistic technology change (OTEC) as the second factor should not be decomposed. Therefore, OMPI can be decomposed as bellow:

$$OMPI = (OPEC \times OSEC) \times OTEC \quad (23)$$

Regarding the above OMPI decomposition, we can be written OPEC as bellows:

$$OPEC_1 = \frac{D_1^{t+1}(x^{t+1},z^{t+1})}{D_1^{t}(x^{t},z^{t})} \quad (24)$$

$$OPEC_2 = \frac{D_2^{t+1}(z^{t+1},y^{t+1})}{D_2^{t}(z^{t},y^{t})} \quad (25)$$

$$OPEC_O = \frac{D_O^{t+1}(x^{t+1},y^{t+1})}{D_O^{t}(x^{t},y^{t})} \quad (26)$$

The OSEC component should be written both CRS and VRS technologies as follows:

$$\text{CRS: } \left[\frac{D_C^{t}(x^{t+1},y^{t+1})}{D_C^{t+1}(x^{t},y^{t})} \frac{D_C^{t+1}(x^{t+1},y^{t+1})}{D_C^{t}(x^{t},y^{t})}\right]^{1/2} \quad (27)$$

$$\text{VRS: } \left[\frac{D_v^{t}(x^{t},y^{t})}{D_v^{t+1}(x^{t+1},y^{t+1})} \frac{D_v^{t+1}(x^{t},y^{t})}{D_v^{t}(x^{t+1},y^{t+1})}\right]^{1/2} \quad (28)$$

Hence, the OSEC component for the two-stage process can be written as bellows:

$$OSEC_1 = \left[\frac{D_{1c}^{t}(x^{t+1},z^{t+1})}{D_{1c}^{t+1}(x^{t},z^{t})}\,\frac{D_{1c}^{t+1}(x^{t+1},z^{t+1})}{D_{1c}^{t}(x^{t},z^{t})}\right]^{1/2} \times \left[\frac{D_{1v}^{t}(x^{t},z^{t})}{D_{1v}^{t+1}(x^{t+1},z^{t+1})}\,\frac{D_{1v}^{t+1}(x^{t},z^{t})}{D_{1v}^{t}(x^{t+1},z^{t+1})}\right]^{1/2} \quad (29)$$

$$OSEC_2 = \left[\frac{D_{2c}^{t}(z^{t+1},y^{t+1})}{D_{2c}^{t+1}(z^{t},y^{t})}\,\frac{D_{2c}^{t+1}(z^{t+1},y^{t+1})}{D_{2c}^{t}(z^{t},y^{t})}\right]^{1/2} \times \left[\frac{D_{2v}^{t}(z^{t},y^{t})}{D_{2v}^{t+1}(z^{t+1},y^{t+1})}\,\frac{D_{2v}^{t+1}(z^{t},y^{t})}{D_{2v}^{t}(z^{t+1},y^{t+1})}\right]^{1/2} \quad (30)$$

$$OSEC_O = \left[\frac{D_{oc}^{t}(x^{t+1},y^{t+1})}{D_{oc}^{t+1}(x^{t},y^{t})}\,\frac{D_{oc}^{t+1}(x^{t+1},y^{t+1})}{D_{oc}^{t}(x^{t},y^{t})}\right]^{1/2} \times \left[\frac{D_{ov}^{t}(x^{t},y^{t})}{D_{ov}^{t+1}(x^{t+1},y^{t+1})}\,\frac{D_{ov}^{t+1}(x^{t},y^{t})}{D_{ov}^{t}(x^{t+1},y^{t+1})}\right]^{1/2} \quad (31)$$

Regarding the above OMPIs in 20, 21 & 22 equations, OMPI>1 demonstrates productivity progress, OMPI=1 indicates constant productivity and OMPI<1 represents productivity decline.

The second components that measure the technological change are as follows:

$$OTEC_1 = \left[\frac{D_{1}^{t}(x^{t+1},z^{t+1})}{D_{1}^{t+1}(x^{t+1},z^{t+1})}\,\frac{D_{1}^{t}(x^{t},z^{t})}{D_{1}^{t+1}(x^{t},z^{t})}\right]^{1/2} \quad (32)$$

$$OTEC_2 = \left[\frac{D_{2}^{t}(z^{t+1},y^{t+1})}{D_{2}^{t+1}(z^{t+1},y^{t+1})}\,\frac{D_{2}^{t}(z^{t},y^{t})}{D_{2}^{t+1}(z^{t},y^{t})}\right]^{1/2} \quad (33)$$

$$OTEC_O = \left[\frac{D_{O}^{t}(x^{t+1},y^{t+1})}{D_{O}^{t+1}(x^{t+1},y^{t+1})}\,\frac{D_{O}^{t}(x^{t},y^{t})}{D_{O}^{t+1}(x^{t},y^{t})}\right]^{1/2} \quad (34)$$

It must be mentioned that OMPI, OPEC and OTEC factors have been constructed under VRS technology, but the OSEC factor has been combined with CRS and VRS assumptions. Similarly, we can use the above equations (20~34) for measuring the pessimistic MPI (PMPI).

Afterward, for Combination of OMPI and PMPI, we use aggregate MPI (AMPI) to make the results from the integrated optimistic and pessimistic MPIs by the geometric mean of the consistent OMPI and PMPI values as bellows:

$$AMPI = [OMPI.PMPI]^{1/2} \quad (35)$$

$$APEC = [OPEC.PPEC]^{1/2} \quad (36)$$

$$ASEC = [OSEC.PSEC]^{1/2} \quad (37)$$

$$ATEC = [OTEC.PTEC]^{1/2} \quad (38)$$

4 Empirical Illustration

Our data are drawn from the financial statements of aforementioned banks for 2009-2013. The data are approximately for the end of March for each year which coincides with the end of the Iranian fiscal year for 5 banks in the abovementioned period. The decline in each bank reflects continuing consolidation in the Iranian banking industry.

Given the financial crisis since late 2012 in Iran which started due to the end of the previous government and country's political situation and also some other reasons at that time, the importance of accounting the banks performance under the most updated scientific models in two different points of view (optimistic and pessimistic) without taking the internal relations within the banks as a black box by the use of network system has become clear. In the traditional DEA models, we have to take the internal relations within the DMUs as a black box and consequently, we cannot discover the causes of internal inefficiency (efficiency). A fortiori, the accuracy and validity of the efficiency measurement via conventional DEA models is less than the network DEA models.

According to the Barros et al. (2009), empirical studies apply two approaches to measure bank outputs and costs. The production approach considers that banks produce accounts of various sizes by processing deposits and loans, incurring in capital and labor costs. The intermediation approach considers banks as transforming deposits and purchased funds into loans and other assets. These two approaches have been applied in different ways depending on the availability of data and the purpose of the study. We define inputs and outputs as following:

The inputs (x_{ij}), intermediate measures (z_{dj}) and outputs (y_{rj}) data are provided in Tables 1 to 5, where five commercial banks (Mellat, Saderat, Sina, Pasargad, Eghtesad Novin) as the DMUs to be evaluated. In this connection, we have considered that the physical assets (PA), the number of employees (NE), deposits value (DV) and the operational costs (OC) are as the whole process inputs, similarly, received commissions (RC), loan payments (LP) and investment amount (IA), are the three intermediary measures in a two-stage process and finally, whole process output is the net revenue (NR). It should be noted that all the data presented in tables below are based on the published reports from the independent auditor and legal inspector of the banks; furthermore, all digits (except NE) are billion Rials.

Table 1
Data Set for Five DMUs with Four Inputs, Three Intermediate Measures and One Output in 2009

DMU	Inputs				Intermediate measures			Output
	PA	NE	DV	OC	RC	LP	IA	NR
Mellat	13.979	24737	386.262	17.827	3.505	1.234	4.563	3.770
Saderat	21.819	29218	324.713	22.083	3.125	6.430	14.654	3.813
Sina	745	1561	30.315	1.063	59	2.112	3.148	700
Pasargad	2.986	4067	105.121	19.415	1.389	400	1.116	3.109
Eghtesad N.	2.640	2693	96.417	2.875	771	276	646	2.150

Source: Research findings.

Table 2
Data Set for Five DMUs with Four Inputs, Three Intermediate Measures and One Output in 2010

DMU	Inputs				Intermediate measures			Output
	PA	NE	DV	OC	RC	LP	IA	NR
Mellat	13.979	24737	386.262	17.827	3.505	1.234	4.563	3.770
Saderat	23.330	29379	410.007	24.226	3.875	14.009	26.621	7.391
Sina	969	1721	41.848	1.543	186	1.109	4.399	1.118
Pasargad	5.769	4531	136.769	30.638	1.910	1.473	2.954	5.924
Eghtesad N.	2.753	2970	115.640	3.286	767	276	1.200	3.003

Source: Research findings.

Table 3
Data Set for Five DMUs with Four Inputs, three Intermediate Measures and One Output in 2011

DMU	Inputs				Intermediate measures			Output
	PA	NE	DV	OC	RC	LP	IA	NR
Mellat	22.293	23014	558.787	34.153	6.578	2.118	20.852	8.067
Saderat	25.458	33856	570.490	30.309	5.512	3.808	21.487	5.111
Sina	1.687	2264	55.928	1.826	406	1.462	7.594	1.706
Pasargad	10.872	5708	166.091	37.674	2.429	2.086	6.298	9.522
Eghtesad N.	2.990	3907	152.071	3.167	989	474	2.773	4.490

Source: Research findings.

Table 4
Data Set for Five DMUs with Four Inputs, Three Intermediate Measures and One Output in 2012

DMU	Inputs				Intermediate measures			Output
	PA	NE	DV	OC	RC	LP	IA	NR
Mellat	37.815	22495	826.116	37.747	5.221	2.190	26.945	15.159
Saderat	64.766	33079	523.476	34.391	4.843	1.147	21.863	7.888
Sina	1.847	2238	76.531	1.845	548	4.476	7.621	4.840
Pasargad	22.584	6720	227.412	38.818	4.996	593	4.987	13.558
Eghtesad N.	4.324	3861	194.576	4.524	1.207	1.882	3.293	4.401

Source: Research findings.

Table 5
Data Set for Five DMUs with Four inputs, Three Intermediate Measures and One Output in 2013

DMU	Inputs				Intermediate measures			Output
	PA	NE	DV	OC	RC	LP	IA	NR
Mellat	43.025	22157	926.408	48.854	13.778	3.321	32.391	21.978
Saderat	69.991	32713	637.692	38.565	5.223	1.271	30.299	9.888
Sina	2.102	2374	93.866	2.404	733	753	6.922	2.592
Pasargad	51.127	7758	294.406	59.465	6.238	2.129	6.948	18.143
Eghtesad N.	3.967	4096	253.493	4.713	1.652	8.765	7.437	5.396

Source: Research findings.

Regarding Table 6, OMPI>1 demonstrates productivity improvement (progress), OMPI=1 indicates constant productivity and OMPI<1 represents productivity decline (regress) which is contrary to the pessimistic point of view as shown in Table 7.

As it is shown, optimistic productivity changes in considered banks during the analyzed period in Table 6 on the average column, indicates that each bank has on the average OMPI>1 signifying a productivity change over the analyzed period. The OMPI was further decomposed in OPEC, OSEC and OTEC. According to the Barros et al. (2009), the change in the technical efficiency score is defined as the diffusion of best-practice technology in the management of the activity and is attributed to investment planning, technical experiences, and management and organization in the banks.

Table 6
The Optimistic DEA-based MPI Values for the Iranian Commercial Banks

DMU	2009-2010				2010-2011			
	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$
Mellat	1.30	1.00	0.89	1.47	0.94	1.00	0.71	1.33
Saderat	1.57	1.00	0.90	1.74	0.51	1.00	0.56	0.91
Sina	0.85	1.00	1.00	0.85	0.93	0.98	1.00	0.94
Pasar.	1.22	1.00	0.94	1.30	1.02	1.00	0.91	1.12
Eght. N.	1.03	1.00	1.00	1.03	1.00	1.00	1.00	1.00
DMU	2011-2012				2012-2013			
	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$
Mellat	1.71	1.00	1.08	1.59	0.93	1.00	0.82	1.15
Saderat	2.34	0.61	1.76	2.19	1.14	1.51	1.00	0.76
Sina	1.67	1.02	1.00	1.67	0.65	1.00	1.00	0.65
Pasar.	1.44	1.00	0.93	1.54	0.66	1.00	0.93	0.70
Eght. N.	0.81	1.00	1.00	0.81	0.93	1.00	1.00	0.93
DMU	Average							
	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$				
Mellat	1.18	1.00	0.86	1.37				
Saderat	1.21	0.98	0.97	1.27				
Sina	0.97	1.00	1.00	0.97				
Pasar.	1.04	1.00	0.93	1.12				
Eght. N.	0.94	1.00	1.00	0.94				

Source: Research findings.

We use GAMS (Generalized Algebraic Modeling System) to estimate each bank's efficiency (inefficiency) measures to generalize to productivity measurement.

5 Application to the Iranian Commercial Banks

In this section, we apply the proposed method to measure and analyze the productivity changes of five Iranian commercial banks over the five years period (2009-2013).

In this case, first of all, for each DMU, we run DEA models (8~19) to calculate MPI's distance functions for two individual stages and overall process and then measure the OMPI values for the two-stage process by equations (20)-(22). Then, we can measure the OPEC, OSEC and OTEC for the two-stage process by equations (24~26), (29~31) and (32~34), respectively. Similarly, we can calculate the PMPI values and subsequently, measure the PPEC, PSEC and PTEC for a two-stage process. Finally, we use AMPI to make the results from the integrated MPI of the optimistic and pessimistic point of views by the geometric mean of the consistent OMPI and PMPI values. In other words, for achieving the final conclusion, we measure the AMPI, AEC and ATC, respectively.

As it is clear in Table 6 from the optimistic DEA point of view, productivities of all DMUs (except Sina) improved during 2009-2010 and productivity growth rates for the DMUs are 30.2% for Mellat, 56.99% for Saderat, -15.1% for Sina, 22.24% for Pasargad and 2.73% for Eghtesad N., respectively. It is clear that Second DMU (Saderat) achieved the greatest productivity progress with 56.99% increase in productivity, while third DMU (Sina) exhibited the most productivity decline with 15.1% decrease in productivity. Similarly, we can analyze the productivity changes of all DMUs during 2010-2013.

Table 7

The Consistent Pessimistic DEA-based MPI Values with Optimistic Concept for the Iranian Commercial Banks

DMU	2009-2010				2010-2011			
	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$
Mellat	Inf.	1	Inf.	Inf.	Inf.	1	Inf.	Inf.
Saderat	Inf.	1	Inf.	Inf.	Inf.	1	Inf.	Inf.
Sina	0.4929	1	1.150	0.43	1.0	1	1.11	0.8648
Pasar.	0.2168	1	1.35	0.16	Inf.	1	Inf.	Inf.
Eght. N.	0.72	1.08	1.09	0.62	0.80	0.71	1.12	1.01
DMU	2011-2012				2012-2013			
	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$
Mellat	Inf.	1	Inf.	Inf.	Inf.	1	Inf.	Inf.
Saderat	Inf.	1	Inf.	Inf.	Inf.	1	Inf.	Inf.
Sina	Inf.	1	Inf.	Inf.	Inf.	1	Inf.	Inf.
Pasar.	Inf.	1	Inf.	Inf.	Inf.	1	Inf.	Inf.
Eght. N.	1.32	1.48	1.71	0.22	1.02	0.86	1.09	1.09
DMU	Average							
	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$				
Mellat	Inf.	1	Inf.	Inf.				
Saderat	Inf.	1	Inf.	Inf.				
Sina	Inf.	1	Inf.	Inf.				
Pasar.	Inf.	1	Inf.	Inf.				
Eght. N.	0.69	0.99	1.12	0.62				

Source: Research findings.

Moreover, on average, the most annual productivity improvement is related to the Saderat Bank which results from the annual pure efficiency change (2.25% decline), scale efficiency change (2.78% regress) and technology change (27.10% decline) during the years under review. It must be noted that these evaluation conclusions are correct, but only from the optimistic point of view.

As we can see in Table 7, from the consistent pessimistic DEA point of view with the optimistic concept, all of the DMUs (except Eghtesad N.) have been infeasible in this case study. As the same way, during 2009-2011 the last DMU is faced negative growth in productivity, but during the 2011-2013 has been achieved positive productivity growth which is 31.60% and 2.34%, respectively. Furthermore, on average, the productivity of the last DMU has declined (-30.8%). Similarly, these conclusions are correct only from the pessimistic point of view.

Table 8

The Aggregate DEA-based MPI Values for the Iranian Commercial Banks

DMU	**2009-2010**				**2010-2011**			
	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$
Mellat	Inf.	1	Inf.	Inf.	Inf.	1	Inf.	Inf.
Saderat	Inf.	1	Inf.	Inf.	Inf.	1	Inf.	Inf.
Sina	0.65	1.00	1.07	0.60	0.94	0.99	1.05	0.90
Pasar.	0.51	1.00	1.13	0.46	Inf.	1.00	Inf.	Inf.
Eght. N.	0.86	1.04	1.04	0.80	0.90	0.84	1.06	1.00
DMU	2011-2012				2012-2013			
	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$
Mellat	Inf.	1	Inf.	Inf.	Inf.	1	Inf.	Inf.
Saderat	Inf.	0.778	Inf.	Inf.	Inf.	1.23	Inf.	Inf.
Sina	Inf.	1.01	Inf.	Inf.	Inf.	1	Inf.	Inf.
Pasar.	Inf.	1	Inf.	Inf.	Inf.	1	Inf.	Inf.
Eght. N.	0.56	1.22	1.08	0.43	0.98	0.93	1.04	1.01
DMU	Average							
	$OMPI_O$	$OPEC_O$	$OSEC_O$	$OTEC_O$				
Mellat	Inf.	1.00	Inf.	Inf.				
Saderat	Inf.	0.99	Inf.	Inf.				
Sina	Inf.	1.00	Inf.	Inf.				
Pasar.	Inf.	1.00	Inf.	Inf.				
Eght. N.	0.8066	1.00	1.06	0.77				

Source: Research findings.

For making a correct and final conclusion, we should integrate the optimistic and pessimistic values via geometric mean to reflect the productivity changes of the DMUs thoroughly. To achieve this target and in order to produce an integrated MPI for the five DMUs, Table 8 displays the aggregate DEA-based MPI values, namely AMPI, for the five DMUs by considering both the optimistic and the pessimistic DEA points of view simultaneously. Specially, productivity growth rate of all DMUs (except Eghtesad N.) have been infeasible (at least for two periods) during the four time periods (2009~2013). As clearly is shown in Table 8, the AMPIs for last DMU during the 2009-2013 were declining to -13.85% in 2009–2010, -10.37% in 2010–2011, -43.86% in 2011-2012 and -2.36% in 2012-2013. The annual average productivity growth rate for last DMU during 2009–2013 was -19.34%. Similarly, we can analyze the other DMUs in the aforesaid time periods.

6 Conclusions

The current paper develops relational models for measuring the total efficiency for the whole process of a two-stage process unit in two different DEA points of view, this is because the conventional DEA uses only optimistic DEA models for efficiency measurement and also the traditional DEA-based MPI uses optimistic DEA models for productivity measurements. Therefore, the results only reflect the productivity changes from the optimistic point of view. Subsequently, we measured the MPI’s distance functions for two individual stages and the whole process for both DEA different points of view by the traditional DEA models and supposed relational models in output oriented BCC models, respectively. In addition, in the DEA standard models, we have to take the DMUs as black boxes and therefore we have not been able to find the main reasons of inefficiency (efficiency). A fortiori, the accuracy and validity of the efficiency measurement via conventional DEA models is less than the network DEA models. Therefore, the identified double frontiers two-stage DEA is more realistic, comprehensive, accurate and validate than the conventional optimistic or pessimistic (or both of them) DEA-based MPI individually. In this paper, in order to develop and modify the previous studies, we have proposed a method to modify Wang and Lan (2011) aggregate MPI and also extend it to a two-stage process which we refer to as the double frontiers two-stage DEA or DFTDEA for measuring the MPI from double frontiers (optimistic and pessimistic) in two-stage DEA simultaneously, and develop the aforementioned approach and related models. The DFTDEA-based MPI considers not only the optimistic productivity changes of DMUs but also determines their pessimistic changes and, not only the shifts of efficiency frontiers but also the movements of inefficiency frontiers. Therefore, it is more practical, useful and applicable than the conventional DEA-based MPI.

The proposed models have been applied to analyze the productivity changes of the Iranian commercial banks. The application results have shown that the OMPI values are different from those measured from the PMPI and cannot be disregarded.

The achieved AMPIs have been applied to analyze the productivity changes of the five Iranian commercial banks during 2009-2013. The results have distinctly proven our expected outcomes and shown that the MPI values measured from the optimistic and pessimistic DEA points of view completely difference.

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