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Original Research Article

Dynamic Cross Hedging Effectiveness between Gold and Stock Market Based on Downside Risk Measures: Evidence from Iran Emerging Capital Market

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This paper examines the hedging effectiveness of gold futures for the stock market in minimizing variance and downside risks, including value at risk and expected shortfall using data from the Iran emerging capital market during four different sub-periods from December 2008 to August 2018. We employ dynamic conditional correlation models including VARMA-BGARCH (DCC, ADCC, BEKK, and ABEKK) and copula-GARCH with different copula functions to estimate volatilities and conditional correlations between Iran gold futures contract return and Tehran stock exchange main index return. The empirical results reveal that the dynamic conditional correlations switch between positive and near-zero values over the period under study. These correlations are high and positive during the major national currency devaluation and are low near to zero during other times. Out-of-sample one-step-ahead forecasts based on rolling window analysis show that DCC and ADCC multivariate GARCH models outperform other models for variance reduction, while a more interesting finding is that the copula-GARCH model outperforms other models for downside risks reduction.

Keywords: Cross Hedging, Iran Emerging Capital Market, Multivariate GARCH, Copula, Downside Risk.
JEL Classification*:* C58, G10, G11, G15.

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1 Introduction

The Iran capital market is an emerging market that has developed with regard to increasing members, the number of listed companies, trading volume, and

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market capitalization to GDP during the last decade. Despite these quantitative developments but this market is still suffering from a lack of various derivative financial instruments like futures contracts on an index or single stock, and so investors are limited in terms of market risk management. One of the rare derivative instruments in Iran's capital market that created more than one decade ago is the gold futures contract. The relationship between gold price and equity market in developed countries has been studied in many pieces of research, and the role of gold as an alternative investment and haven asset vs. traditional asset class has almost been proven. In the case of Iran as a developing economy, the relationship between gold and equity prices, which both are traded with national currency is different, because commodity exporter who has a positive earning relationship with national currency devaluation are dominant in terms of market capitalization in Iran. So equity market of Iran naturally should have some positive correlation with the domestically traded gold price during national currency devaluation periods and maybe a low correlation during periods in which national currency is stable.

Cross hedging is one of the important topics in financial risk management literature, the basic concept of cross hedging is about using futures contract of an asset to hedge the market risk related to another asset. The determination of the optimal hedge ratio, defined as the ratio between number of futures position's to the per-unit spot position, importantly influences hedged portfolios' performance.

One of the arising literature in determining optimal hedge ratio is minimizing downside risk measure instead of traditional risk measures like variance. Using variance as target risk measure to determine optimal hedge ratio may not achieve to downside risk reduction if the utility function of hedgers is not quadratic or if asset returns are not elliptically distributed.

Value-at-Risk (VaR) and Expected Shortfall (ES) are two of the most well-known downside risk measures used in risk hedging with futures. Value at risk (VaR) is a statistic that quantifies the extent of possible financial losses within a portfolio, or position over a specific time frame. Additionally, ES as a measure of risk, complementing, and in parts substituting, the more-familiar VaR measure. Expected Shortfall is the expected return on an asset conditional on the return being below a given quantile of its distribution, namely its VaR. ES has long been known to be a coherent measure of risk (Artzner, et al. 1999).

Multivariate Generalized Autoregressive Conditional Heteroscedasticity (MGARCH) models are commonly used to capture the time-varying nature of covariance matrix and the correlation between spot and futures returns of

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different asset classes and thus estimate the associated optimal hedge ratio. Various multivariate GARCH type models, besides the conditional mean dynamics models like VARMA are employed for hedge ratio calculation. Most of the studies in comovement and hedging has used models like BEKK (Baba, Engle, Kraft, and Kroner, 1990), DCC (dynamic conditional correlation, Engle, 2002) or their asymmetric form, namely ABEKK and ADCC, which allows taking into account asymmetry in the dynamic of markets and financial time series. Most of the studies in comovement literature focus on the dependecy between returns using the Pearson correlation. A limitation of the Pearson correlation is that it is based on the assumption of normality. More precisely, it is only a measure of dependence in the elliptical family of distributions (Zhao and Goodwin, 2012). This limitation amplifies in solving the minimum-downside risk hedge ratios, which requires the estimation of the entire joint distribution of spot and futures price movements and their tail dependency. One way to overcome the Pearson correlation shortage is to apply copulas to measure the dependence structure between asset returns.

Copula dependency functions have been broadly employed as an effective tool to model comovement and tail dependency between. Copulas provide a flexible, modular possibility for constructing multivariate distributions. The copula-based GARCH model has recently shown its efficiency to capture time-varying characteristics of the variables in interest.

This paper seeks to use from Iranian gold futures contract to minimize market risk (including variance, VaR, and ES) of equity during four different sub-periods according to the national currency movement direction. The one-step-ahead rolling window scheme will employed to construct optimal hedge ratios. We apply the Copula-GARCH model with two elliptical functions (Gaussian and Student's-t) and two Archimedean functions (Frank and Gumbel) and compare their hedging effectiveness with four well-known multivariate GARCH type models, namely BEKK, DCC, ABEKK, and ADCC.

This paper makes several contributions to the literature. First, our study is novel in that, as far as we know, this is the first time that pair of tow asset class’s gold-stock dependency is examined in Iran as an emerging market. We examine the dynamic correlation between gold and stock market and the performance of use of gold as a hedging asset against stock; this helps to introduce the gold futures contract of Iran's capital market to potential investors, especially foreign investors, and its capability to manage equity risk during national currency devaluation time. Second, while many researchers

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choose a particular MGARCH model (e.g., DCC-GARCH) and then present results without providing a comparison of how their model compares to another, this study will make comprehensive comparison the optimal hedge ratios obtained from BEKK and DCC and their asymmetric form namely ABEKK and ADCC models and also with those obtained from copula-GARCH models with different copula functions. It provides a complete understanding of how hedging effectiveness varies between different multivariate GARCH and copula-GARCH models. Univariate and multivariate GARCH models are applied to obtain one-step-ahead hedge ratios. The one-step-ahead daily optimal hedge ratios are obtained using a rolling window scheme to estimate all parameters dynamically. Third, this study not just concentrates on the minimum-variance hedging strategy like many other pieces of research, we also estimate models to minimize downside risk measure for 95, 99, and 99% VaR and ES. Using copula type models helps to better capture the joint distribution of gold-stock movements and their tail dependency, which is required for more effective downsides risk minimization.

The remainder of this paper is organized as follows. Section 2 presents relevant literature. Section 3 presents the methodology used in this paper. Section 4 describes the data and the preliminary statistics. Section 5 discusses the multivariate GARCH models' estimation results, empirical results of the models, and methods. Finally, Section 6 summarizes the main conclusions.

2 Relevant Literature

We recognize that different types of volatility dynamic conditional correlation models are broadly applied to capture volatility of various asset class comovement since dynamic hedging stock market with gold for minimizing variance, VaR, and ES is our main focus in this research. We discuss a short review of relevant researches that focus on similar dynamic models or hedging stock with gold and other related commodities.

Baur and McDermott (2010) investigate gold as a hedge asset for equity markets of some emerging markets. Using descriptive and econometric analysis for a sample spanning 30 years from 1979 to 2009, they conclude that gold is both a hedge and a safe haven for major European stock markets and the US but not for Australia, Canada, Japan, and large emerging markets such as the BRIC countries.

Baur and McDermott (2010) investigate gold as a hedge asset for equity markets of some emerging markets. Using daily data from November 1995 to

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November 2010, they find that gold acts as a weak safe haven and a strong hedge asset.

Gurgun and Unalmiş (2014) examine role of gold for hedging equity market of some developing economics. According to their findings gold has a hedge and safe haven role. They conclude that gold is both a hedge and a safe haven for domestic investors in most of these countries. This result also holds in the post-2008 crisis period. Besides, when falls in equity markets become more severe, gold acts as a safe haven in a larger set of countries for both domestic and foreign investors.

Sadorsky (2014) uses VARMA-AGARCH and DCC-AGARCH models to investigate the multivariate volatility between emerging market equity, oil prices, copper prices, and wheat prices. He finds Correlations between these assets increased considerably after 2008, and on average, oil provides the cheapest hedge for emerging market stock prices

Arouri et al. (2015), using the VAR-GARCH models over March 22, 2004, through March 31, 2011, examine the return and volatility spillovers between the stock market and gold prices in China. Several competing for multivariate volatility models which are commonly used in the finance literature (CCC–GARCH, DCC–GARCH, diagonal BEKK–GARCH, scalar BEKK–GARCH, and full-BEKK–GARCH) are also for comparison purpose in their research. One part of their findings indicate adding gold to stocks reduces risk and make better hedging against equity risk.

Basher and Sadorsky (2016) employ the DCC, ADCC, and GO-GARCH to model conditional correlations between emerging market stock prices, oil prices, VIX, gold prices, and bond prices using daily data. They find that the ADCC model's Hedge ratios are most effective to variance reduction for hedging emerging market equity prices with using pair asset including oil, VIX, or bonds.

Chkili (2016) employs the ADCC model to examine the dynamic relationships between gold and BRICS countries' stock markets using data set period covers the period from January 2000 to July 2014. He finds that the time-varying correlation between gold and stock markets is near zero to negative during financial crisis. His evidence shows that adding gold to a portfolio leads to a risk reduction.

Chen, Zheng, and Qu (2020) use VAR-ABEKK and DCC to examine comovement between international crude oil, new energy, and rare earth markets in China. They find that cumulative risk between oil and new energy markets indirectly transfer.

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In recent research of hedging, VaR and CVaR have been applied for the risk reduction target. For example, Chang (2011) employs bivariate Markov regime Switching Autoregressive Conditional Heteroscedastic (SWARCH) model to formulate the optimal VaR hedging strategy to minimize the downside risk of the hedged position Using Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) futures data.

Cao, Harris, and Shen (2010) present minimum-VaR and minimum-CVaR hedge ratios based on the Cornish-Fisher expansion of the quantile of the hedged portfolio return distribution. Using spot and futures FTSE daily returns from February 1994 to August 2008, they find that the semiparametric approach is superior to the standard minimum-variance approach, and to provides a greater reduction in both VaR and CVaR.

Ubukata (2018) employs DCC models under multivariate skewed t-distributions to assess the incremental value of dynamic futures hedging models in minimizing downside risk measures. Using spot and futures returns of the Nikkei index from March 11, 1996, to March 1, 2013, he concludes the unconditional minimum downside risk approaches underperform than the novel conditional approaches.

Karmakar & Sharma (2020) propose a hybrid GO-GARCH-EVT-copula model to estimate minimum quantile risk hedge ratios. They examine the proposed model's hedging effectiveness compared to three other competing models using thirty-five pairs of daily spot and futures price series data from various stock, currency, and commodity markets across the world. The evidence suggests that the proposed combined approach performs best in estimating minimum quantile risk hedge ratios.

Jondeau and Rockinger (2006) employ the Copula-GARCH model of conditional dependencies to describe dependency between European markets using sample covers the period from January 1, 1980, to December 29, 2000, they find the dependency increase substantially in the same direction in a crash or a boom market.

Lai, W.S. Chen, Gerlach (2009) employ a time-varying Copula-TGARCH model to evaluate the optimal hedge ratio in five East Asian markets using data from January 1, 1998 to June 10, 2005, they show hedge ratios obtained from Copula-TGARCH have hedging efficiency.

Zhao and Goodwin (2012) apply Copula-GARCH model in spot and futures agricultural products to obtain conditional correlation and the cross hedge ratio. They find that performance of Copula-GARCH models has effectiveness as a cross hedging mechanism.

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Barbi and Romagnoli (2014) present copula based approach for estimating minimum downside-risk hedge ratios employing data for the main UK and US indices and EUR/USD and EUR/GBP exchange rates from January 1, 1990, to January 31, 2011. They indicate that their copula based approach outperforms nonparametric approach in downside risk reductions.

Jammazi and Reberdo (2016) employ wavelets and copulas to examine oil-stock dependence structure at different time scales using using different static and time-varying copula functions for data covering the period January 4, 2000, to June 29, 2015. Their findings show diversification benefits and downside risk reductions in short time scale for this tow asset class. Kotkatvuori-Örnberg (2016) uses the Copula DCC-EGARCH model to investigate the efficiency of the futures hedge implemented through the currency markets. The estimation period covers the return series from January 14, 2000, to December 27, 2013. Also, for artificial data, the bootstrap method for data simulation is utilized. They show that the inclusion of the external realized variance estimators into the variance equation effectively reduces the variance of the in hedging portfolios.

Sukcharoen and Leatham (2017) examine the use of a vine copula approach to estimate multiproduct hedge ratios that minimize the refinery's downside risk. They find that the D-vine copula model is an effective choice in managing the refinery's downside risk.

Tiwari et al. (2019) examine the time-varying correlations between S&P 500 index and six cryptocurrencies, namely, Ripple, Dash, Stellar, Litecoin, Ethereum, and Bitcoin, using a copula-ADCC-EGARCH model. Using daily closing prices from August 7, 2015, to June 15, 2018, they find their time-varying correlations are very low.

3 Methodology

The basic concept of hedging is to reduce market risk of a spot position (S_t) by applying futures contracts (F_t)). Let h_t represent the short position taken in the futures market at time t under a particular hedging strategy, The return on the hedged portfolio over a period, R_{ht} , is given by:

$$R_{ht} = R_{st} + hr_t * R_{ft} \quad (1)$$

where R_{st} and R_{ft} are the spot and futures returns at time t, respectively, and hr_t is the hedge ratio. The optimal hedge ratio for minimizing the variance of hedged portfolio return given by:

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$$hr_t^* = -\frac{Cov(R_{st}, R_{ft})}{Var(R_{ft})} = -\rho_{s,f} * \sqrt{\frac{Var(R_{st})}{Var(R_{ft})}} \quad (2)$$

Where Var is the variance operator, Cov is the covariance operator, and ρ is the correlation coefficient operator.

The degree of hedging effectiveness, proposed by Ederington (1979), is measured by the percentage reduction in hedged position variance. Therefore, the degree of hedging effectiveness, denoted as HE, can be expressed as follows:

$$\text{HE} = \frac{Var(R_{st}) - Var(R_{ft})}{Var(R_{st})} = 1 - \frac{Var(R_{ft})}{Var(R_{st})} \quad (3)$$

The concept of variance reduction as a measure of hedging effectiveness can be extended for downside risk gains, which are also evaluated by computing the percentage reduction in hedged position's VaR and ES with respect to the unhedged position like Equation (3).

3.1 The Bivariate GARCH Approach

The BGARCH volatility model can be written as follows::

$$R_t = \begin{bmatrix} R_{st} \\ R_{ft} \end{bmatrix} = \mu_t + u_t, \;\; u_t | F_{t-1} \sim F(0, H_t) \quad (4)$$

where $R_t, \mu_t = (\mu_{st}, \mu_{ft})', u_t = (u_{st}, u_{ft})'$ are vector-valued conditional mean functions, F denotes a bivariate distribution, and H_t is a time-varying 2 × 2 positive definite conditional covariance matrix. The above model can be estimated by a maximum likelihood estimation. In this paper, VARMA (vector autoregressive moving average) model uses as conditional mean models of the BGARCH model, and Bayesian information criterion (BIC) determines the number of lags length.

We consider the different types of BGARCH models in this paper. The above model can be estimated by a maximum likelihood estimation. To guarantee the positive-definite constraint, Engle and Kroner (1995) propose the Baba-Engle-Kraft-Kroner (BEKK) model:

$$H_t = C'C + A' u_{t-1} u_{t-1}' A + B' H_{t-1} B \quad (5)$$

Where *A*, *B*, and *C* are 2 × 2 matrices, but *C* is upper triangular matric. In reduced form of this model the matrices *A* and *B* are assumed to be diagonal,

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thus forming a diagonal BEKK model. The parameters of matrices specified as follows:

$$C = \begin{bmatrix} c_{SS} & c_{sf} \\ 0 & c_{ff} \end{bmatrix}, A = \begin{bmatrix} a_{SS} & 0 \\ 0 & a_{ff} \end{bmatrix}, B = \begin{bmatrix} b_{SS} & 0 \\ 0 & b_{ff} \end{bmatrix}$$

This model allows for dynamic dependence between the volatility series. On the other hand, the model has several disadvantages. First, the parameters in A and B do not have direct interpretations concerning lagged values of volatilities or shocks. Second, the number of parameters employed increases rapidly, and limited experience shows that many of the estimated parameters are statistically insignificant, introducing additional modeling complications (Tsay, 2010). Another shortcoming of this parameterization is that it restricts the covariance dynamics to be governed by the product of corresponding parameters of the two variance equations (Bauwens et al., 2006).

The asymmetric version of the BEKK(1,1) model, introduced by Grier et al. (2004), as follows

$$H_t = C'C + A'u_{t-1}u_{t-1}'A + G'z_{t-1}z_{t-1}'G + B'H_{t-1}B \quad (6)$$

z_{t-1} is the 2×1 vector in Bivariate case. For covering asymmetric effect in multivariate case, when $u_{t-1} \leq 0$ it means negative shock, then $z_{t-1} = u_{t-1}$, elsewhere, $z_{t-1} = 0$. The matrix G measures the asymmetric effects between 2 variables and has form like the matrices of A and B.

The conditional correlations can vary over time because the conditional volatility updates the conditional correlations. One of the most famous models in the literature is DCC (Dynamic Conditional Correlation) proposed by Engle (2002).

DCC takes into account the time-varying correlation instead of constant correlation assumption. Moreover, in contrast to BEKK model, number of parameters in DCC model increases linearly rather than exponentially, DCC model is estimated in two steps. First, the univariate GARCH model estimated. Second, the conditional correlations are estimated:

$$H_t = D_t R_t D_t \quad (7)$$

R_t is the conditional correlation matrix, and D_t is a diagonal matrix with conditional standard deviations on its diagonal. In the case of bivariate hedging, we have $i = 1, 2$ or $i = s, f$:

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$$D_t = diag(\sqrt{h_{i,t}}) = diag(\sqrt{h_{s,t}}, \sqrt{h_{f,t}}) \quad (8)$$

$$R_t = (diag(\sqrt{q_{i,t}}))^{-1} Q_t (diag(\sqrt{q_{i,t}}))^{-1} \quad (9)$$

h obtained using univariate GARCH model, in case of the GARCH(1,1):

$$h_{i.t} = c_i + a_i u_{i,t-1}^2 + b_i h_{i,t-1} \quad (10)$$

Q_t is a symmetric positive definite matrix.

$$Q_t = (1 - v_1 - v_2)\bar{Q} + v_1 \varepsilon_{t-1} \varepsilon_{t-1}' + v_2 Q_{t-1} \quad (11)$$

In the bivariate case, $\bar{Q}$ is the 2 × 2 unconditional correlation matrix of the standardized residuals ε_{it} ($\varepsilon_{it} = u_{i,t}/\sqrt{h_{i,t}}$). The parameters $v_1$and v_2 non negative. The $v_1 + v_2 < 1$ ensures that model is mean-reverting.

Cappiello et al. (2006) present an asymmetric version of DCC, namely ADCC based on DCC and GARCH model of Glosten et al. (1993):

$$h_{i,t} = c_i + a_i u_{i,t-1}^2 + b_i h_{i,t-1} + g_i u_{i,t-1}^2 \mathrm{I}(u_{i,t-1}) \quad (12)$$

$\mathrm{I}(u_{i,t-1})$ is indicator function and equals one if $u_{i,t-1} \leq 0$ and 0 elsewhere. When g be positive, it means that negative residuals tend to increase the volatility more than positive residuals. For the ADCC model:

$$Q_t = (\bar{Q} - A'\bar{Q}A + G'\bar{Q}^{-}G + B'\bar{Q}B) + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + G'\varepsilon_{t-1}^{-}\varepsilon_{t-1}^{-}{}'G + B'Q_{t-1}B \quad (13)$$

A, *B* and *G* are 2 × 2 parameter matrices and ε_t^- are zero threshold standardized errors which is ε_t when less than zero and zero elsewhere. $\bar{Q}$ and $\bar{Q}^-$ are the unconditional matrices of ε_t and ε_t^- respectively.

3.2 Copula-GARCH Approach

A copula function represents a flexible dependence structure between random variables. In statistical terms, a copula1 is a multivariate function with uniform marginals that represents the dependence structure among random variables. By Sklar's theorem, for a 2-dimensional joint distribution function $G(x, y)$ with continuous marginal cumulative density functions (CDF) $F_X(x), F_Y(y)$ and, there exists a copula function $C(\cdot)$ that satisfies:

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$$G(x, y) = C(F_X(x), F_Y(y)) \quad (14)$$

According to above Equation, dependence structure of copula separated from marginal distributions. In our case R_{st} and R_{ft} are two random variables denoting spot and futures returns at period t, and their conditional CDF are given by $\mathrm{u}_t = F_t^s(R_{st}|I_{t-1}), \mathrm{v}_t = F_t^f(R_{ft}|I_{t-1})$ which are distributed as continuous uniform variables on (0, 1); where I_{t-1} denotes all past returns of R_{st}, R_{ft} at time $t-1$. so the conditional copula function $C_t(\mathrm{u}_t, \mathrm{v}_t|I_{t-1})$ is defined by the time-varying CDF of spot and futures. It is possible to estimate marginal densities ($\mathrm{f}_t^s, \mathrm{f}_t^f$) and the copula density (c_t) distributions parameters using the log-likelihood function. Inference for the margins (IFM) for obtaining copula parameters is an approach to estimate marginal distributions and the copula separately, this approach solves dimension curse of estimation all parameters in one step. We estimate the model and distribution parameters using the log-likelihood function.

In this paper Copula-GARCH model formed at two steps like Lai et al. (2009) or Zhao and Goodwin (2012) approach: The first-step marginal normal ARMA(1,1)-GARCH(1,1) parameter estimates provide estimated values of standardized residuals ε_{it} ($\varepsilon_{it} = u_{i,t}/\sqrt{h_{i,t}}$). Standardized residuals obtained in step one used in the second-step to form copula dependence structure between R_{st} and R_{ft}. In this paper, we apply two elliptical copulas (Gaussian and Student's-t) and two Archimedean functions (Frank and Gumbel) to measure the dependence between R_{st} and R_{ft}.

The Gaussian copula has no tail dependence, while the T copula allows different degrees of symmetric tail dependence (DoF). As the DoF increase, the T copula converges to the Gaussian copula.

$$C_t^G(\mathrm{u}_t,\mathrm{v}_t|\rho_t) = \int_{-\infty}^{\Phi^{-1}(\mathrm{u}_t)} \int_{-\infty}^{\Phi^{-1}(\mathrm{v}_t)} \frac{1}{(2\pi\sqrt{|R_t|})} \exp\left(\frac{U\prime R_t^{-1} U}{2}\right) dndm \quad (15)$$

Where $\mathrm{u}_t = F_t^s(\varepsilon_{st}|I_{t-1}), \mathrm{v}_t = F_t^f(\varepsilon_{ft}|I_{t-1}), U = [m\ n]'$, $\Phi^{-1}(\cdot)$ is the inversed standard normal CDF, and R_t is the correlation matrix$\begin{pmatrix} 1 & \rho_t \\ \rho_t & 1 \end{pmatrix}$.

In Student-t copula, the dependence structure introduces an additional parameter, which is the degree of freedom (DoF):

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$$C_t^T(\mathrm{u}_t,\mathrm{v}_t|v,\rho_t) = \int_{-\infty}^{\mathrm{T}^{-1}(\mathrm{u}_t)} \int_{-\infty}^{\mathrm{T}^{-1}(\mathrm{v}_t)} \frac{\Gamma(\frac{v+2}{2})}{\Gamma(\frac{v}{2})\left(\sqrt{(\pi v)^2|R_t|}\right)\left[1+\frac{U\prime R_t^{-1}U}{v}\right]^{\frac{v+2}{2}}} dndm \quad (16)$$

where $\Gamma(\cdot)$ is Gamma function .

In contrast to elliptical copulas, Archimedean copulas can only be used to measure positive dependence and allow asymmetric tail dependence in some cases. This study employs the Gumbel(Gu) and Frank(F) Archimedean copulas:

$$C_t^{Gu}(\mathrm{u}_t,\mathrm{v}_t|\theta_t) = \exp(-((-\log(\mathrm{u}_t))^{\theta_t} + (-\log(\mathrm{v}_t))^{\theta_t})^{1/\theta_t}) \quad (17)$$

$$C_t^F(\mathrm{u}_t,\mathrm{v}_t|\theta_t) = -\frac{1}{\theta_t}\log\left\{1 - \frac{(1-\exp(-\theta_t \mathrm{u}_t))(1-\exp(-\theta_t \mathrm{v}_t))}{1-\exp(-\theta_t)}\right\} \quad (18)$$

3.3 Estimating Conditional Minimum Downside Risk Hedge Ratios

In this paper, we estimate conditional hedge ratio to minimize a VaR and ES at different confidence levels, in contrast to minimum variance hedge ratio with an analytical expression according to Equation (2), for VaR and ES minimization, there is no clear analytical expression. We generate one-step-ahead spot and futures returns through Monte Carlo simulations and Using a grid search approach to calculate the corresponding hedge ratio that minimizes VaR and ES of the hedged portfolio returns (Conlon and Cotter, 2013; Ubukata, 2018). Thus, we calculate the conditional hedge ratios for VaR and ES minimizing with a daily rebalancing in 5 steps as follows:

1) Estimate the parameters of VARMA-BGARCH or copula-GARCH models using the first 80% of in-sample data of the past daily spot and futures returns $(1,2.\cdots t)$.
2) Calculate one-day-ahead forecasts of the conditional mean vector and covariance matrix (μ_{t+1}, H_{t+1}) VARMA-BGARCH or copula-GARCH models.
3) Simulate correlated spot and futures returns with n=10,000 number of simulated sample paths (realizations) by using μ_{t+1}, H_{t+1} through Monte Carlo simulations.
4) Applying a range of alternative hedge ratios to find the hedge ratio that minimize VaR and ES of hedged portfolio according to grid search approach.
5) Adding most recent observation and removing oldest observation, and repeating step 1–4.

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4 Data

This paper uses the Tehran Stock Exchange (TSE) major index and gold futures contract. We shape the gold futures price series as follows. First, we specify the nearby futures contract as the contract with the nearest active trading delivery month to the day of trading. We use the prices of the nearby futures contract until the contract reaches the first day of the delivery month or its expiry date as well as the prices of the next nearby contract. When the futures contract reaches the first day of the delivery month, we switch the nearby contract prices, which may affect the volatility of the prices.

We obtain the TSE index series from the Tehran Stock Exchange and gold futures from Iran Mercantile Exchange (IME). For the robustness of out of sample hedging performance, we split the data into four different sub-periods according to the positive relationship of Iran equity market and gold price during national currency devaluation periods and maybe low correlation during periods when the national currency is stable. The considered sub-periods with different statistical correlation amount of spot and futures returns are as follows: (P1) the first period: December 6, 2008, to August 17, 2011; (P2) the second period: August 20, 2011, to March 25, 2013; (P3) the third period: March 26, 2013, to October 8, 2017; and (P4) the fourth period: October 9, 2017, to August 29, 2018. Periods 2 and 4 are national currency devaluation periods. In contrast, the national currency is relatively stable during periods 1 and 3. The regulatory body of Tehran capital market has stopped gold futures contract trades temporary since end of August, 2018 because of arising some non-proven concerning remarks about the role of gold futures market as an inflationary signal leading to Iran's traditional currency market. The last period has less than 1 year (210 observations), reducing the robustness of estimations and results in the fourth period. It is one of the limitations of this research in the Iran capital market. We use the last 20% observations of each sub-period for out-of-sample evaluating.

Table 1 gives the descriptive statistics for the gold-stock returns in four sub-periods. The kurtosis of index and gold futures returns are larger than 3, indicating that both return series are leptokurtic. The Jarque–Bera test show that variables have not normal distribution. The estimated standard deviations of gold futures return during national currency devaluation (P2 and P4) are about twice larger than those in other sub-periods. It shows that gold futures returns are more volatile during national currency devaluation. The Pearson linear correlation coefficient and its p-value pointed to positive and significant dependence between the index level and gold price during national currency devaluation, not correlated during period 1 and little negative and significant

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during period 3; thus, we except hedging effectiveness of gold futures contract for equity market will be higher during national currency devaluation in Iran.

Table 1
Descriptive statistics for gold-stock returns in four sub-periods.

	Period 1		**Period 2**		**Period 3**		**Period 4**		**Total data**	
	Index	gold	Index	gold	Index	gold	Index	gold	Index	gold
Sample size	650		377		1095		210		2332	
Mean	0.002	0.001	0.001	0.002	0.001	-0.001	0.002	0.005	0.001	0.001
Median	0.001	0.001	0.000	0.001	0.000	0.000	0.001	0.007	0.000	0.000
Mode	0.000	-0.038	-0.019	-0.083	-0.055	-0.072	-0.025	-0.095	0.000	-0.095
Max	0.054	0.046	0.035	0.098	0.036	0.051	0.072	0.094	0.072	0.098
Min	-0.025	-0.038	-0.019	-0.083	-0.055	-0.072	-0.025	-0.095	-0.055	-0.095
Std.dev.	0.007	0.011	0.007	0.024	0.007	0.012	0.010	0.023	0.876	0.133
skewness	0.61	0.05	0.56	0.09	0.28	0.01	2.21	-0.35	0.01	0.02
kurtosis	9.61	5.42	4.65	4.30	9.42	10.70	15.43	6.09	11.50	8.80
JB test	1223.28	158.68	62.36	26.87	1897.43	2708.02	1523.48	88.03	7310.83	3273.46
p-Value	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
correlation	0.0479		0.4404		-0.1005		0.3782		0.1625	
p-Value	0.223		0.000		0.001		0.000		0.000	

Notes: The four sub-periods including; Period 1: December 6, 2008, to August 17, 2011, with 650 observations; Period 2: August 20, 2011, to March 25, 2013, with 377 observations; Period 3: March 26, 2013, to October 8, 2017, with 1095 observations; Period 4: October 9, 2017, to August 29, 2018, with 210 observations Jarque and Bera (JB) normality test statistic follows a χ^2distribution with 2 degrees of freedom.
Source: Research Findings

5 Empirical Hedging Performance Results

5.1 Model Estimation Results

We obtain the optimal conditional hedge ratio by maximizing the log-likelihood of the VARAM-BGARCH and copula-Garch models. We consider the simplest conventional BGARCH model with conditional bivariate normal error distribution for our empirical study. For a conditional mean model of BGARCH models, we use VARAM(p,q) model and select the optimal lags order using the Bayesian information criterion (BIC) to select the degrees p and q. For the conditional mean and variance model of copula-BGARCH models, we use ARMA(1,1)-GARCH(1,1) model with conditional univariates normal error distribution. In order to incorporate the information in the error terms, we estimate the VARMA-BGARCH model through a two-stage procedure. More specifically, we save the estimated residuals from the

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VARMA and estimate the BGARCH model using the residual series in the second stage.[1]

The estimation of the multivariate GARCH and copula parameters by in-sample data are presented in Table 2, Table 3, and Table 4.

The estimation of the BEKK and ABEKK models parameters by in-sample data are presented in Table 2, almost all of ARCH and GARCH parameters are significant. The sum of the estimates ${a_{ii}}^2 + {b_{ii}}^2 \; for \; i = s, f$ is close to unity for gold futures return in all periods except period 2, for equity index return is high in all periods, showing high persistence in spot-futures volatility except period 2 for gold. Asymmetric parameters are just significant during period 3; thus, BEKK and ABEKK shouldn't have significant hedging effectiveness differences in rest periods.

Table 2

Estimation results for BEKK and ABEKK bivariate GARCH models in four sub-periods.

	Period 1		**Period 2**		**Period 3**		**Period 4**	
	BEKK	ABEKK	BEKK	ABEKK	BEKK	ABEKK	BEKK	ABEKK
c_{ss}	0.0029*	0.0029**	0.0024***	0.0024***	0.0024***	0.0023***	0.0020**	0.0020***
	(0.0511)	(0.0463)	(0.0000)	(0.0010)	(0.0079)	(0.0055)	(0.0120)	(0.0087)
c_{sf}	0.0000	0.0000	0.0227***	0.0227***	0.0000	-0.0001	0.0034**	0.0092
	(0.7477)	(0.5014)	(0.0000)	(0.0000)	(0.8950)	(0.7090)	(0.0184)	(0.1659)
c_{ff}	0.0019***	0.0019***	0.0000	0.0000	0.0016***	0.0018***	0.0084***	0.0188**
	(0.0000)	(0.0000)	(0.9957)	(0.9257)	(0.0018)	(0.0012)	(0.0000)	(0.0151)
a_{ss}	0.3464**	0.3465**	0.3806***	0.3806***	0.3628***	0.3740***	0.7880***	0.7854***
	(0.0154)	(0.0139)	(0.0000)	(0.0100)	(0.0000)	(0.0000)	(0.0015)	(0.0015)
a_{ff}	0.3714***	0.3714***	-0.1036*	-0.1036*	0.2626***	0.2237***	-0.0524	-0.1435
	(0.0000)	(0.0000)	(0.0915)	(0.0615)	(0.0000)	(0.0001)	(0.4073)	(0.1425)
g_{ss}		0.0000		0.0000***		-0.0752		0.0692
		(0.7364)		(0.0010)		(0.2234)		(0.4704)
g_{ff}		0.0000		0.0000***		0.2496***		0.2635
		(0.2755)		(0.0010)		(0.0024)		(0.3405)
b_{ss}	0.7615***	0.7612***	0.8135***	0.8135***	0.8708***	0.8668***	0.6156***	0.6171***
	(0.0021)	(0.0017)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0058)	(0.0038)
b_{ff}	0.9057***	0.9057***	-0.3249***	-0.3249***	0.9566***	0.9501***	0.9131***	-0.2574
	(0.0000)	(0.0000)	(0.0014)	(0.0013)	(0.0000)	(0.0000)	(0.0000)	(0.7263)
Total LL	3768.522	3768.52	1863	1863	5869.21	5875.59	1068.94	1069.589

Notes: The maximum likelihood estimates (p-values) of unknown parameters for the BEKK and ABEKK models are presented (in parentheses). LL stands for log-likelihood. In this table, as well as all the following ones, the significance is denoted by superscripts at the 1% (***), 5% (**), and 10% (*) levels.

Source: Research Findings

[1] To save space, we do not report the VARMA and ARMA-GARCH models estimation results. These can be obtained from the authors upon request.

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The estimation of the DCC and ADCC models parameters by in-sample data are presented in Table 3; almost of ARCH and GARCH parameters are significant; the sum of the estimates $a_i + b_i \; for \; i = s, f$ is close to unity for equity index return in all periods, showing high persistence in spot-future volatility, for gold futures return is not high during periods 2 and 4. Univariate Asymmetric GJR parameters are just significant and low negative during periods 2 and 3; thus, there is no leverage effect in univariate volatility models. The parameter *v*2 is significant and large in all periods except period 3, which strongly suggests the persistence of the time-varying nature of the correlation between stock-gold returns. The asymmetric parameter *g* isn't significant and large in all periods except period 1. Different leverage effects may arise from different arbitrage activities, heterogeneity, asymmetric information, or/and contract liquidity.

Table 3
Estimation results for DCC and ADCC bivariate GARCH models in four sub-periods.

	Period 1		**Period 2**		**Period 3**		**Period 4**	
	DCC	ADCC	DCC	ADCC	DCC	ADCC	DCC	ADCC
c_s	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
a_s	(0.1706) 0.1745**	(0.2566) 0.2419*	(0.1016) 0.1572**	(0.1422) 0.1737*	(0.1556) 0.1569***	(0.1738) 0.2405***	(0.2646) 0.6109*	(0.3612) 0.7805*
g_s	(0.0338)	(0.0745) -0.1710	(0.0462)	(0.0822) -0.1103**	(0.0000)	(0.0005) -0.1442**	(0.0596)	(0.0936) -0.4793
b_s	0.4832*	(0.1625) 0.5806**	0.7214***	(0.0368) 0.7775***	0.7480***	(0.0426) 0.7466***	0.3889	(0.1981) 0.4590
c_f	(0.0986) 0.0000**	(0.0393) 0.0000*	(0.0000) 0.0005***	(0.0000) 0.0006***	(0.0000) 0.0000	(0.0000) 0.0000	(0.1136) 0.0005***	(0.1455) 0.0005***
a_f	(0.0294) 0.1777***	(0.0957) 0.1991***	(0.0000) 0.0000	(0.0000) 0.0000	(0.1465) 0.0902**	(0.1301) 0.0793**	(0.0000) 0.0376	(0.0000) 0.0000
g_f	(0.0008)	(0.0001) -0.0787	(0.9982)	(1.0000) 0.0000***	(0.0136)	(0.0339) 0.0259	(0.3829)	(1.0000) 0.1073
b_f	0.7910***	(0.3005) 0.8125***	0.1131**	(0.0001) 0.0042***	0.8963***	(0.5229) 0.8942***	0.0000	(0.3643) 0.0000
q	(0.0000) 0.0108	(0.0000) 0.0154	(0.0135) 0.3518***	(0.0000) 0.3576***	(0.0000) -0.1085***	(0.0000) -0.1102***	(0.7141) 0.2064***	(0.9972) 0.2092***
v_1	(0.8051) 0.0131	(0.8249) 0.0000	(0.0000) 0.0000	(0.0000) 0.0000	(0.0026) 0.0064	(0.0024) 0.0027	(0.0039) 0.0000	(0.0035) 0.0000
g	(0.2972)	(1.0000) 0.0784**	(1.0000)	(1.0000) 0.0001	(0.4619)	(0.8057) 0.0070	(1.0000)	(1.0000) 0.0000
$v2$	0.9376***	(0.0307) 0.8980***	0.9636***	(0.9979) 0.9628***	0.0910	(0.5684) 0.0000	0.9651***	(1.0000) 0.9659***
Total LL	(0.0000) 3778.7	(0.0000) 3785.1	(0.0000) 1864.9	(0.0000) 1867	(0.6106) 5887.6	(0.9999) 5893.2	(0.0000) 1069.2	(0.0000) 1072

Notes: The maximum likelihood estimates (p-values) of unknown parameters for the DCC and ADCC models are presented (in parentheses). LL stands for log-likelihood. In this table as well as all the following ones, the significance is denoted by superscripts at the 1% (***), 5% (**), and 10% (*) levels.
Source: Research Findings

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The copula parameters' estimation by in-sample data is presented in Table 4; All different copula family estimates are significant except period 1 which the correlation between two assets is very low.

Table 4
Estimation results for copula parameters model in four sub-periods.

	Period 1	**Period 2**	**Period 3**	**Period 4**
		Gaussian (or normal)		
ρ_t	0.0538	0.3461***	-0.1127*	0.2275**
	(0.2130)	(0.0010)	(0.0581)	(0.0451)
Total LL	1897.019	1160.726	3186.266	680.722
		Student-t		
ρ_t	0.0539	0.3486***	-0.1270**	0.2266**
	(0.2082)	(0.0011)	(0.0412)	(0.0458)
v	4674301	8.6399***	4.7670*	49.0782**
	(0.2320)	(0.0010)	(0.0589)	(0.0448)
Total LL	1897.575	1161.482	3186.812	681.278
		Frank		
θ_t	0.2610	2.3163**	-0.8099*	1.4167**
	(0.2230)	(0.0150)	(0.0670)	(0.0495)
Total LL	1895.209	1159.176	3184.496	679.259
		Gumbel		
θ_t	1.0000	1.2957***	1.0000*	1.1619**
	(0.2401)	(0.0020)	(0.0651)	(0.0496)
Total LL	1893.950	1157.924	3183.396	678.050

Notes: The maximum likelihood estimates (p-values) of unknown parameters for the copula joint dependency are presented (in parentheses). LL stands for log-likelihood. In this table, as well as all the following ones, the significance is denoted by superscripts at the 1% (***), 5% (**), and 10% (*) levels.
Source: Research Findings

5.2 Hedging Performance

To conduct the in-sample and out-of-sample analysis, we split the sample in each of the sub-periods into an in-sample with 80% of data and an out-of-sample with 20% of remaining data. We reestimate the models' parameters with a daily rollover keeping a fixed sample size of 80% of data. This rollover method continues for all out-of-sample periods in each sub-period.

Table 5 summarizes the results of in-sample variance reduction. As shown clearly, the gold futures contract was useful in variance reducing of stock in national currency devaluation periods (periods 2 and 4). The low correlation of gold-stock in national currency stable periods leads gold as a useless instrument to hedge the stock market in Iran. Among different models, DCC and ADCC, like Copula-GARCH models, have better hedging effectiveness

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compared to weak results of BEKK and ABEKK models during periods 2 and 4.

Table 5
The in sample Variance reduction effectiveness for gold-stock in four sub-periods.

Model type		Period 1	Period 2	Period 3	Period 4
VARMA-BGARCH	BEKK	-0.036	0.150	-0.051	0.020
	DCC	-0.002	0.191	0.004	0.099
	ABEKK	-0.036	0.150	-0.030	0.022
	ADCC	0.001	0.197	0.002	0.101
copula-GARCH	Gaussian	-0.002	0.186	0.002	0.123
	T	-0.002	0.191	-0.003	0.127
	Frank	-0.001	0.187	0.002	0.116
	Gumbel	0.000	0.195	-0.004	0.128

Notes: This table reports the in sample hedging performance of the time-varying hedge ratios estimated by the VARMA-BGARCH and copula-GARCH models in four sub-periods. The measure of hedging performance is variance reduction.
Source: Research Findings

Table 6 and 7 summarize the results of in-sample VaR and ES reduction at the 95%, 97.5%, and 99% confidence level. As shown clearly, gold futures contract was useful in downside risk-reducing of stock in national currency devaluation periods (periods 2 and 4) and a little during period 3, the low dependency of gold-stock in national currency stable periods leads gold as a useless instrument to hedge the downside risk of the stock market in Iran. Copula-GARCH models have a little bit better performance among different models than DCC and ADCC models, and BEKK and ABEKK have the worst result at all.

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Table 6
The in sample Value-at-Risk reduction effectiveness for gold-stock in four sub-periods.

Model type		Period 1			Period 2			Period 3			Period 4		
	VaR Confidence Level(%)	95	97.5	99	95	97.5	99	95	97.5	99	95	97.5	99
VARMA-BGARCH	BEKK	-0.010	-0.050	-0.150	0.049	0.055	0.052	0.027	-0.016	-0.041	-0.097	0.127	0.025
	DCC	0.010	-0.026	-0.004	0.050	0.072	0.063	0.014	0.062	0.039	-0.116	0.129	0.185
	ABEKK	-0.010	-0.050	-0.150	0.049	0.055	0.052	0.023	-0.075	-0.044	-0.108	0.116	0.035
	ADCC	-0.001	-0.012	-0.050	0.041	0.055	0.062	0.009	0.071	0.048	-0.105	0.120	0.143
copula-GARCH	Gaussian	-0.002	0.000	-0.005	0.044	0.055	0.063	0.009	0.040	0.040	-0.096	0.112	0.196
	T	-0.003	-0.001	-0.005	0.045	0.058	0.065	0.009	0.052	0.031	-0.100	0.101	0.203
	Frank	-0.009	-0.008	-0.003	0.044	0.053	0.063	0.009	0.042	0.038	-0.087	0.107	0.184
	Gumbel	0.000	0.001	0.000	0.054	0.089	0.066	-0.001	-0.018	-0.009	-0.101	0.100	0.204

Notes: This table reports the in sample hedging performance of the time-varying hedge ratios estimated by the VARMA-BGARCH and copula-GARCH models in four sub-periods. The measure of hedging performance is 95%, 97.5% and 99% VaR reduction.
Source: Research Findings

Table 7
The in sample Expected Shortfall reduction effectiveness for gold-stock in four sub-periods.

Model type		Period 1			Period 2			Period 3			Period 4		
	ES Confidence Level (%)	95	97.5	99	95	97.5	99	95	97.5	99	95	97.5	99
VARMA-BGARCH	BEKK	-0.049	-0.049	0.006	0.063	0.060	0.038	-0.033	-0.051	-0.009	0.013	0.015	-0.006
	DCC	-0.006	-0.012	0.003	0.063	0.057	0.038	0.016	0.010	0.022	0.101	0.149	0.181
	ABEKK	-0.049	-0.049	0.006	0.063	0.060	0.038	-0.021	-0.038	-0.002	-0.008	-0.007	-0.060
	ADCC	-0.010	-0.018	0.016	0.059	0.055	0.036	0.018	0.012	0.022	0.091	0.128	0.157
copula-GARCH	Gaussian	-0.006	-0.005	0.004	0.064	0.057	0.044	0.013	0.008	0.021	0.107	0.169	0.196
	T	-0.005	-0.005	0.003	0.064	0.055	0.041	0.014	0.004	0.016	0.112	0.178	0.199
	Frank	-0.003	-0.002	0.002	0.064	0.057	0.044	0.013	0.007	0.021	0.097	0.154	0.190
	Gumbel	0.000	0.000	0.000	0.064	0.054	0.037	-0.006	-0.005	-0.005	0.113	0.179	0.200

Notes: This table reports the in sample hedging performance of the time-varying hedge ratios estimated by the VARMA-BGARCH and copula-GARCH models in four sub-periods. The measure of hedging performance is 95%, 97.5%, and 99% ES reduction.
Source: Research Findings

Panel A of Table 8 presents the out-of-sample time-varying hedging ratios for variance minimization. Average hedge ratios during period 1 are near zero, which accommodates the insignificant correlation between two assets during this period. Average hedge ratios during periods 2, 4 are negative and relatively high with respect to a cross hedge between two different asset classes. Average hedge ratios during period 3 are positive and relatively low.

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It accommodates a negative correlation between two assets at this period. Almost average hedge ratios obtained from DCC and ADCC are near to copula- GARCH models and higher from BEKK and ABEKK in all periods.

Table 8
Summary statistics of average time-varying minimum variance hedge ratios and variance reduction in four sub-periods.

Model type		**Period 1**	**Period 2**	**Period 3**	**Period 4**
Panel A: Average out of sample optimal hedge ratio for variance minimization in four sub-periods.					
VARMA-BGARCH	BEKK	0.008	-0.071	0.037	-0.028
	DCC	0.004	-0.127	0.065	-0.170
	ABEKK	0.004	-0.100	0.029	-0.056
	ADCC	0.004	-0.136	0.067	-0.170
copula-GARCH	Gaussian	-0.013	-0.122	0.068	-0.149
	T	-0.013	-0.124	0.072	-0.149
	Frank	-0.018	-0.120	0.061	-0.131
	Gumbel	0.001	-0.126	0.000	-0.164
Panel B: The out of sample variance reduction effectiveness for gold-stock in four sub-periods.					
Model type		Period 1	Period 2	Period 3	Period 4
VARMA-BGARCH	BEKK	-0.017	0.209	-0.043	0.151
	DCC	-0.008	0.245	0.024	0.239
	ABEKK	-0.006	0.208	-0.044	0.122
	ADCC	-0.008	0.256	0.013	0.267
copula-GARCH	Gaussian	0.002	0.186	0.016	0.229
	T	0.002	0.242	0.019	0.222
	Frank	0.002	0.236	0.021	0.206
	Gumbel	-0.001	0.243	0.002	0.241

Notes: This table reports the out-of-sample hedging performance of the time-varying hedge ratios estimated by the VARMA BGARCH and copula GARCH models in four sub-periods. Panel A reports statistics of out-of-sample forecasted hedge ratios. Panel B reports out-of-sample performance of the time varying hedge ratios. The measure of hedging performance is variance reduction.
Source: Research Findings

Panel B of Table 8 presents the out-of-sample time-varying hedging effectiveness for variance minimization. The ADCC outperforms other models during periods 2 and 4, two important periods. The results of DCC are near to ADCC, T family copula and Gumbel copula have variance reduction near the DCC model in sample results, BEKK and ABEKK show weak hedging effectiveness in all periods. The Gaussian copula among different copula families shows weaker results, which accommodates the non-normality of data and fat tail in two returns series. Variance reduction during periods 1 and 3 is low, showing gold has no power to reduce stock variance during national currency stability periods. One interesting practical results

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from overall Panel A and B of Table 8 is that by using the best model (ADCC) during national currency devaluation periods in Iran, we can reduce more than 25% variance of a long stock position by just taking on average between 13% to 17% short position in gold futures contracts (relative to spot position amount).

Panel A of Table 9 presents the out-of-sample time-varying hedging ratios for VaR minimization. Average hedge ratios during period 1 are near zero, and this accommodates the insignificant correlation between two assets during this period. Average hedge ratios during periods 2, 4 are negative and relatively high with respect to a cross hedge between two different asset classes. Average hedge ratios during period 3 are positive and relatively low. It accommodates a negative correlation between two assets during this period. The average minimum VaR hedge ratios do not show a uniform pattern among different models compared to minimum variance hedge ratios. It arises from the reality of no clear analytical expression to obtain a minimum VaR hedge ratio. As we explained in section (3.3), hedge ratios calculation for VaR and ES minimization are based on the Monte Carlo simulations and grid search approach.

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Table 9
Summary statistics of average time-varying minimum VaR hedge ratios and VaR reduction in four sub-periods.

Model type	Confidence Level (%)	Period 1			Period 2			Period 3			Period 4		
		95	97.5	99	95	97.5	99	95	97.5	99	95	97.5	99
Panel A: Average out of sample optimal hedge ratio for VaR minimization in four sub-periods.													
VARMA-BGARCH	BEKK	-0.020	-0.025	-0.019	-0.112	-0.113	-0.112	0.025	0.022	0.019	-0.091	-0.097	-0.092
	DCC	-0.002	0.001	0.001	-0.142	-0.144	-0.140	0.069	0.070	0.066	-0.219	-0.218	-0.232
	ABEKK	-0.022	-0.024	-0.022	-0.114	-0.114	-0.117	0.038	0.040	0.034	-0.092	-0.095	-0.090
	ADCC	-0.002	-0.003	-0.004	-0.149	-0.154	-0.157	0.072	0.069	0.068	-0.243	-0.232	-0.231
copula-GARCH	Gaussian	-0.016	-0.015	-0.015	-0.164	-0.082	-0.082	0.065	0.074	0.062	-0.223	-0.223	-0.223
	T	-0.016	-0.015	-0.013	-0.159	-0.085	-0.106	0.068	0.087	0.076	-0.222	-0.222	-0.222
	Frank	-0.021	-0.020	-0.018	-0.161	-0.107	-0.120	0.055	0.067	0.059	-0.194	-0.196	-0.196
	Gumbel	0.000	0.001	0.001	-0.178	-0.126	-0.144	0.000	0.000	0.000	-0.246	-0.246	-0.246
Panel B: The out of sample Value-at-Risk reduction effectiveness for gold-stock in four sub-periods.													
VARMA-BGARCH	BEKK	0.009	-0.039	0.016	0.187	0.167	0.098	-0.018	-0.014	-0.004	0.185	0.172	0.000
	DCC	0.005	-0.010	0.030	0.190	0.176	0.117	0.041	-0.007	-0.020	0.126	0.090	-0.066
	ABEKK	0.006	-0.038	0.019	0.189	0.167	0.097	-0.006	-0.086	-0.247	0.212	0.141	-0.027
	ADCC	0.009	0.020	0.022	0.209	0.184	0.111	-0.028	-0.007	0.015	0.116	0.081	-0.025
copula-GARCH	Gaussian	0.000	0.001	0.002	0.258	0.231	0.114	-0.064	0.018	0.012	0.147	0.099	-0.034
	T	0.001	0.007	0.000	0.258	0.235	0.113	-0.051	0.002	-0.042	0.161	0.105	-0.030
	Frank	0.001	0.013	-0.004	0.183	0.286	0.246	0.229	0.048	-0.016	0.180	0.119	-0.032
	Gumbel	0.000	0.001	0.000	0.277	0.241	0.229	0.002	0.002	0.006	0.138	0.088	-0.034

Notes: This table reports the out-of-sample hedging performance of the time-varying hedge ratios estimated by the VARMA-BGARCH and copula-GARCH models in four sub-periods. Panel A reports statistics of out-of-sample forecasted hedge ratios. Panel B reports the out-of-sample performance of the time-varying hedge ratios. The measure of hedging performance is 95%, 97.5%, and 99% VaR reduction.
Source: Research Findings

Panel B of Table 9 presents the out-of-sample time-varying hedging effectiveness for VaR minimization. Overall the copula-GARCH models outperform other models during periods 2 and 4, two important periods. Two Archimedean functions copula outperform elliptical functions. Maybe it arises from the reality of better tail dependence covering with Archimedean functions, which is an important factor for hedge ratio estimation based on downside risk minimization. VaR reduction during periods 1 and 3 are low, and these results show gold has no power to reduce VaR of stock during the low correlation time of two assets. One interesting practical results from overall Panel A and B of Table 9 is that by using the best model (Archimedean copula-based models) during national currency devaluation periods in Iran, we can reduce more than 25% VaR (with different confidence levels)of a long stock position by just taking on average between 10% to 17% short position in gold futures contracts (relative to spot position amount), this is an important role from regulatory points of view because it helps the financial institution to

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adjust stock portfolio's VaR to regulatory accepted level with just adding small amount short gold position to portfolio.

Panel A of Table 10 presents the out-of-sample time-varying hedging ratios for ES minimization. The result is very close to VaR downside risk measure; average hedge ratios during period 1 are near zero, and this accommodates the insignificant correlation between two assets during this period. Average hedge ratios during periods 2, 4 are negative and relatively high with respect to a cross hedge between two different asset classes. Average hedge ratios during period 3 are positive and relatively low; this accommodates a negative correlation between two assets during this period. Like average VaR minimization hedge ratios, the average minimum ES hedge ratios do not show a uniform pattern among different models compared to minimum variance hedge ratios. It arises from the reality of no clear analytical expression to obtain a minimum VaR hedge ratio. As we explained in section (3.3), hedge ratios calculation for VaR and ES minimization are based on the Monte Carlo simulations and grid search approach.

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Table 10
Summary statistics of average time-varying minimum ES hedge ratios and ES reduction in four sub-periods.

Model type	Confidence Level (%)	Period 1			Period 2			Period 3			Period 4		
		95	97.5	99	95	97.5	99	95	97.5	99	95	97.5	99
Panel A: Average out of sample optimal hedge ratio for ES minimization in four sub-periods.													
VARMA-BGARCH	BEKK	-0.031	-0.033	-0.026	-0.115	-0.112	-0.116	0.025	0.023	0.023	-0.090	-0.093	-0.085
	DCC	-0.005	-0.004	-0.005	-0.143	-0.142	-0.141	0.072	0.072	0.070	-0.219	-0.238	-0.235
	ABEKK	-0.035	-0.033	-0.023	-0.114	-0.111	-0.112	0.042	0.038	0.037	-0.093	-0.097	-0.102
	ADCC	-0.010	-0.009	-0.008	-0.154	-0.156	-0.152	0.076	0.076	0.072	-0.239	-0.239	-0.231
copula-GARCH	Gaussian	-0.020	-0.018	-0.016	-0.099	-0.088	-0.137	0.048	0.036	0.034	-0.224	-0.224	-0.224
	T	-0.018	-0.018	-0.016	-0.148	-0.161	-0.182	0.067	0.053	0.044	-0.223	-0.223	-0.223
	Frank	-0.025	-0.024	-0.022	-0.154	-0.157	-0.172	0.047	0.041	0.034	-0.196	-0.196	-0.196
	Gumbel	-0.001	-0.001	-0.001	-0.173	-0.173	-0.186	-0.001	-0.001	-0.001	-0.246	-0.246	-0.246
Panel B: The out of sample Expected Shortfall reduction effectiveness for gold-stock in four sub-periods.													
VARMA-BGARCH	BEKK	0.003	0.009	0.015	0.150	0.118	0.078	-0.165	0.001	-0.004	0.138	0.000	0.000
	DCC	0.004	0.010	0.003	0.161	0.133	0.100	0.015	-0.013	-0.013	0.075	-0.066	-0.066
	ABEKK	0.016	0.022	0.015	0.150	0.117	0.077	-0.165	-0.265	-0.508	0.124	-0.028	-0.028
	ADCC	0.014	0.027	0.001	0.168	0.129	0.091	0.012	-0.019	-0.019	0.067	-0.067	-0.067
copula-GARCH	Gaussian	0.001	0.007	-0.001	0.204	0.219	0.211	-0.012	-0.005	0.001	0.087	-0.035	-0.035
	T	0.001	0.003	-0.001	0.235	0.226	0.215	-0.017	-0.001	0.001	0.092	-0.030	-0.030
	Frank	0.001	0.013	-0.001	0.229	0.245	0.213	0.000	0.003	-0.011	0.105	-0.032	-0.032
	Gumbel	0.001	0.003	0.000	0.235	0.244	0.214	0.003	0.004	0.005	0.076	-0.034	-0.034

Notes: This table reports the out-of-sample hedging performance of the time-varying hedge ratios estimated by the VARMA-BGARCH and copula-GARCH models in four sub-periods. Panel A reports statistics of out-of-sample forecasted hedge ratios. Panel B reports the out-of-sample performance of the time-varying hedge ratios. The measure of hedging performance is 95%, 97.5%, and 99% ES reduction.
Source: Research Findings

Panel B of Table 10 presents the out-of-sample time-varying hedging effectiveness for ES minimization. Overall, the copula-GARCH models outperform other models during periods 2 and 4, two important periods. Two Archimedean functions, copula and relatively T copula outperform Gaussian functions; maybe it arises from the reality of better tail dependence covering with these copulas, which is an important factor for hedge ratio estimation based on downside risk minimization. ES reduction during periods 1 and 3 are low, and these results show gold has no power to reduce ES of stock during the low correlation time of two assets. One interesting practical results from overall Panel A and B of Table 10 is that by using the best model (copula-based models) during national currency devaluation periods in Iran, we can reduce more than 21% ES (with different confidence levels)of a long stock position by just taking on average between 14% to 18% short position in gold futures contracts (relative to spot position amount), this is an important role

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from regulatory points of view because it helps the financial institution to adjust stock portfolio's ES to the regulatory desired level with just adding little amount short gold position to portfolio.

6 Conclusions

This paper examines the hedging effectiveness of gold for the stock market in minimizing variance and downside risks, including value at risk (VaR) and expected shortfall (ES) using data from the Iran emerging capital market. We employ dynamic conditional correlation models including VARMA-BGARCH (DCC, ADCC, BEKK, and ABEKK) and copula-GARCH models with two elliptical functions (Gaussian and Student's-t) and two Archimedean functions (Frank and Gumbel) to estimate volatilities and conditional correlations between Iran gold futures contract and Tehran Stock Exchange main Index during four different sub-periods according to the national currency movement direction. The empirical results reveal that the correlations between gold-stock pair is positive during national currency devaluation periods and near-zero or low negative values over other periods. This research considers a conditional downside risk hedging strategy based on the Monte Carlo simulations and grid search approach to minimize the VaR and ES of the hedged portfolio returns. Out-of-sample one-step-ahead forecasts based on rolling window analysis for variance minimization show the ADCC outperforms other models during national currency devaluation periods. The results of the DCC model are near to ADCC. For downside risk minimization, the copula-GARCH models outperform other models during national currency devaluation periods. Copula-GARCH models based on Archimedean functions outperform elliptical functions. It arises from the reality of better tail dependence covering with Archimedean functions, which is an important factor for hedge ratio estimation for downside risk minimization. The empirical results during national currency stability periods in Iran reveal that cross hedging of stock with gold for any target risk measures are useless, and gold has no power to significantly reduce risk measures of the stock portfolio during these periods.

The results of this paper have practical implications for investors and regulatory bodies of financial institutions. First, the stock market and gold display a positive correlation during national currency devaluation periods in Iran; investors can adjust stock portfolio's risk measures to desired levels (according to their personal risk tolerance) by just adding little amount of short gold position to the portfolio. Second, the correlation of stock-gold is low, near-zero during national currency stability periods in Iran, suggesting that

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gold can't act as a safe haven for stockholders in these times. Third, the finding of this paper is in confirmation with acceptable cross hedging effectiveness of gold futures contract for the stock market by multivariate DCC, ADCC, and copula-based models during national currency devaluation periods in Iran, regulatory bodies of financial institutions in Iran should consider this achievement, they can extend a variety of financial derivative instruments and encourage financial institutions to use dynamic, advanced hedging models for risk management.

This study can be extended in several ways. DCC and ADCC models show strength to estimate minimum variance hedge ratios. On the other hand, copula-based models provide quite flexible models of the dependency structure between two financial variables and show strength to estimate minimum downside risk hedge ratios. One can further use different copula-based functions with DCC and ADCC in hybrid models to capture the benefits of two ideas simultaneously in Iran's capital market. Future studies could also consider hedging performances of applied models to other downside risk hedging strategy like the exponential spectral risk measure (ERM) and lower partial moments (LPM) or utility-maximizing hedged portfolio returns.

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